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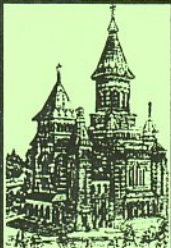
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MATHEMATICAL MODELS APPLIED TO
INVESTIGATE AIRCRAFT SYSTEMS

Written by

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PART II.

DIAGNOSTICS OF SYSTEMS BASED UPON THEIR MATHEMATICAL MODELS

Written by
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THEORETICAL BACKGROUND

9.1 Introduction

The model is a simplified copy of the real system which emphasizes its important characteristics and neglects secondary characters are not determinant from the point of view of the investigation. The abstracted model can be analog, homologue and mathematical one.

The homologue model is similar to the real system geometrically. For example, wind-tunnel models of airfoils or aircraft are homologue models. The analog model does not need to be similar to the real system, but its principle has to be analog with the process occurring on the real modeled system. As a rule, an analog computer means an analog model [27].

9.2 The Mathematical Model

In the engineering practice the most widespread model is the mathematical one. Nowadays, the computer technology and numerical-mathematical methods develop quickly and at the same time diagnostics methods that are based upon the mathematical modeling become important and developing part of the engineering practice.

The mathematical model gives the most concise characterization of an occurrence [4]. The mathematical model is the mathematical equation or system of equations which describes the internal principles of the process occurring on the system from the point of view of the given investigation.

On the basis of the characters of mathematical equations which describe the behavior of the system or their determination, the following mathematical models can be distinguished in pairs [15]:

Static - Dynamic

The mathematical model will be a static one if the state of the system can be described by algebraical equations or differential equations which do not contain derivatives with respect to the time. As a rule, they are called as stationary or steady models.

The dynamic (non-stationary) mathematical models describe the changes of the system parameters depending on the time. They can be vulgar or partial differential equations. It is possible that the changes of the system parameters can

be described by not only on a time interval, but on a transformed domain chosen practically.

Linear - Non-linear

The linear models consist of only variables or their derivatives multiplied - in general stationary - coefficients. A linear mathematical model can be a linear or linearized equation or a system of equations.

The non-linear mathematical models are free from the requirement mentioned above. At least one of the equations which describe process occurs in the system is non-linear.

For simpler solving and investigation, the non-linear mathematical models can be transformed into linear ones (see the Chapter 10.3).

Continuous parameter - Discrete parameter

In case of continuous parameter (continuous time) models the dependent variables can have values at every moment of the investigated time interval continuously.

The mathematical model is a discrete parameter (discrete time) one if its dependent variables can only have some value at certain moments of the investigated time.

As a rule, continuous time processes are modeled by discrete time models when the equations are solved numerically by any time-shifting.

Continuous state-space - Discrete state-space

If the variables can have all value continuously, the mathematical model is called a continuous state-space one.

A mathematical model is considered to be of a discrete state-space one if the values of the dependent variables constitute a count finite or non-finite set.

For easier solving, continuous state-space processes can be transformed to discrete state-space one by discretization.

Deterministic - Stochastic

In case of a deterministic model, the dependencies of output parameters on input parameters can be described unambiguously in some time internal or domain of the independent variables.

Stochastic (random) mathematical models contain random functions which can describe interdependencies between dependent and independent variables of the examined system.

Obviously the list mentioned above of the kinds of the mathematical models is not complete, of course. Mathematical model realized in the engineering practice means the synthesis of the above mentioned ones.

9.3 Creating Mathematical Model

The logic and the type of the modeling are determined by answers for the following question:

✈ What are the main goals of the investigation based on mathematical modeling?

✈ How can the mathematical model be set up?

✈ How can the mathematical model be checked and qualified?

✈ What is the most optimal strategy of the collection of the missing data?

✈ How can the non-linearity be solved?

✈ What are the economic and technical requirements?

✈ Can you experiment with the real system continuously for the checking of the model?

✈ How can the mathematical model be simplified?

✈ What is the most optimal model for the investigation of the real system?

The setting up of a real model can be carried out based upon the logical scheme that figure 9.1 shows. The possible errors are written in italics [25].
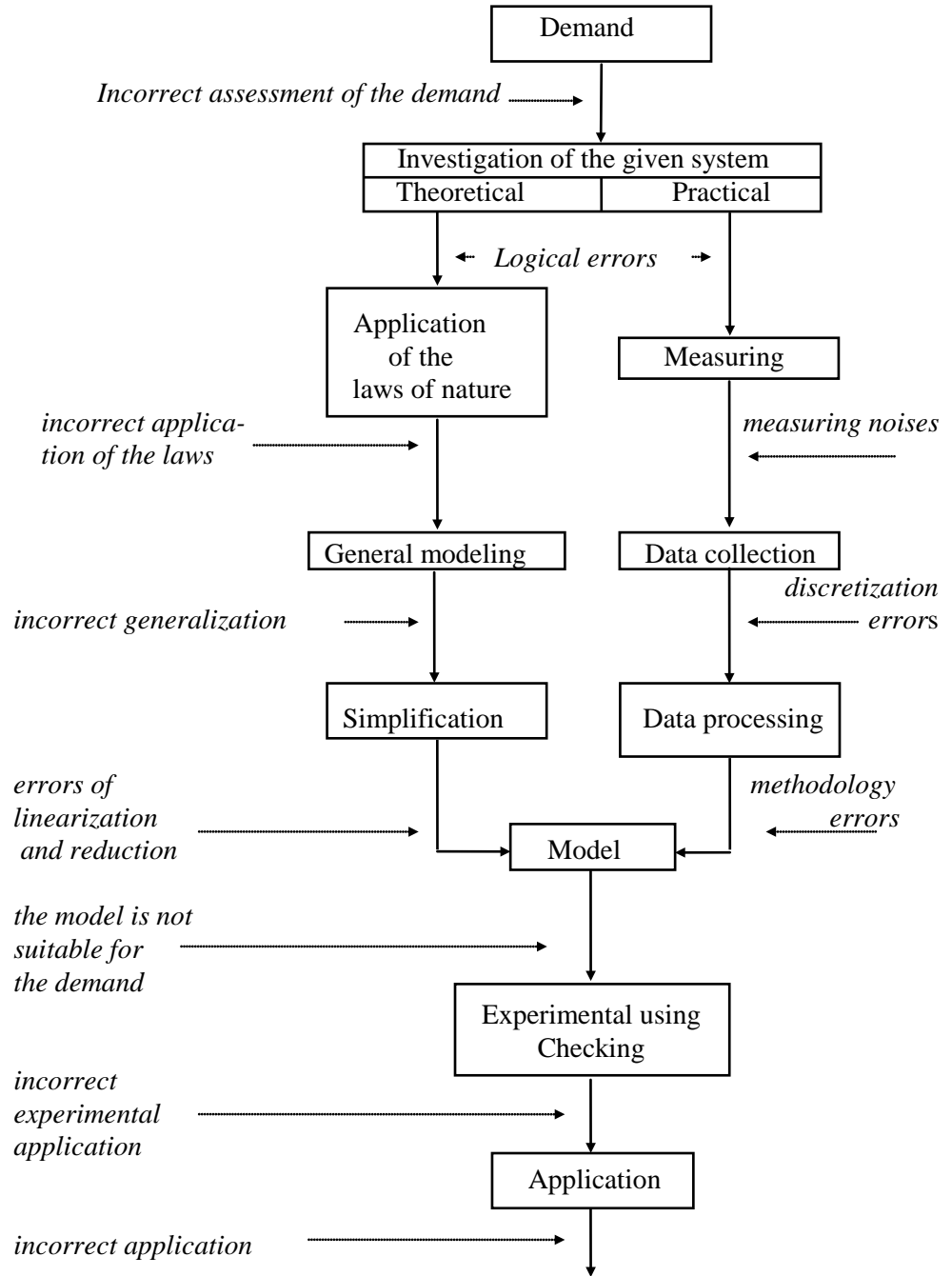
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Fig. 9.1. Logical Scheme of Modeling

METHODOLOGY OF MATHEMATICAL DIAGNOSTIC MODELING

10.1. Introduction

In this chapter the basic methodology of mathematical modeling of technical systems will be shown. Since the author's main goal is to demonstrate it basically for engineers who like to use mathematical models during their work, the method will be shown in the case of the example shown by Figure 10.1.
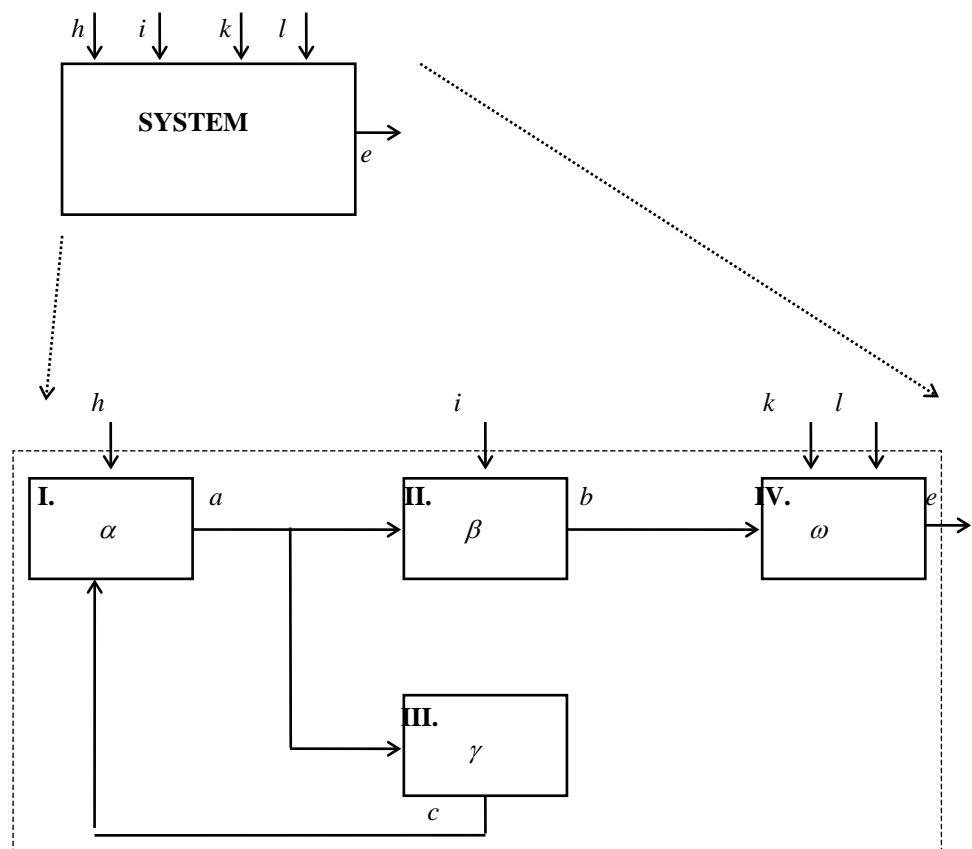
Figure 10.1. Block and Block-Diagram of the System

10.2. The Setting-up of the Mathematical Model

The setting up of mathematical model should start with splitting up the investigated system into its functional units. The Figure 10.1. demonstrates this step of the modeling.

Now it is a very important question that which part of the system is necessary for modeling of its investigated regime. For example in case of a stationary regime of pneumatic and hydraulic systems, the filters are not important equipment. Because of in this case the pressures in the system have been equalized. But during their non-stationary regimes the filters have very important function as chokes which influence the change of pressures in several chambers of the system in the function of time.

These above determined units should be examined and the interdependencies between their input and output parameters should be established mathematically. In the technical practice, the mathematical model can be written basically in two ways:

✈   WHITE BOX METHOD;

The model should be written by analytical equation on the basis of scientific knowledge. In this case you should use physical rules which depict processes occurring in the investigated equipment. Using white box method, you have to know the working principle and the nominal (designed) values of all internal parameters of given system unit.

✈   BLACK BOX METHOD.

The model is written by analyzing of the output parameters responded to the given input ones. This method should be used if the physical processes occurring in the parts of the system and internal structures of equipment is not known. In this case the mathematical model should be set-up by the investigation of the behavior of the real system. For example one of the black box methods is the dimensional analysis.

The equations mentioned above form the mathematical model of the system. For example (for following demonstrations) this model can be written in the case mentioned above, that is the mathematical model of the system see Figure 10.1. :

✈   Equipment I. :

$$a = hc\alpha \tag{10.1}$$

✈   Equipment II. :

$$b = a + i + \beta \tag{10.2}$$

✈   Equipment III. :

$$c = a\gamma^{\kappa} \tag{10.3}$$

✈   Equipment IV. :

$$e = \frac{2kl\omega^{\frac{\kappa-1}{\kappa}}}{b} \tag{10.4}$$

or in a simpler way:

$$f(\underline{y}) = g(\underline{x}) \tag{10.5}$$

where:

$\underline{x}$ - vector of dependent parameters:

$$\underline{x}^T = \begin{bmatrix} a & b & c & e \end{bmatrix} \quad ; \tag{10.6}$$

$\underline{y}$ - vector of independent parameters:

$$\underline{y}^T = \begin{bmatrix} h & \alpha & i & \beta & \gamma & k & l & \omega \end{bmatrix} \quad . \tag{10.7}$$

The elements of the vectors are parameters of the example system - see Figure 10.1. The α ; β ; γ and ω are internal parameters of equipment (for example stiffness of spring or flown area). The a ; b ; c ; e ; h ; i ; k and l are the input and output parameters of the given system and equipment (for instance pressures, mass flow and power).

10.3. Setting-up of the Linear Mathematical Diagnostic Models

For setting up a linear diagnostic model, the mathematical model which is basically a non-linear system of equations should be linearized. For linearization, the following methods can be used:

✈   LOGARITHMIC LINEARIZATION;
✈   DIRECT DIFFERENTIATION;
✈   TAYLOR SERIES;
✈   LIE-MAGNUS SERIES.

In the following only the logarithnic linearization will be depicted in detail, because this method is not well-known.

10.3.1. The Logarithmic Linearization

Using the logarithmic linearization, firstly, the natural logarithm (to e base) of both sides of the general non-linear equation

$$y = f(x_1; x_2; \dots x_n) \quad , \tag{10.8}$$

should be formed:

$$\ln y = \ln f(x_1; x_2; \dots x_n) \quad . \tag{10.9}$$

As the next step, the total differential of the latter one should be formed, using the basic differential quotient of the natural logarithm:

$$(\ln \eta)' = \frac{1}{\eta} \quad , \tag{10.10}$$

and the rule of derivation of the function of functions. We introduce the equation

$$\frac{d\beta_i}{\beta_i} \approx \frac{\Delta\beta_i}{\beta_i} = \delta\beta_i \tag{10.11}$$

and substitute for the equation given above formally. Then you get the equation

$$\delta y = K_1 \delta x_1 + K_2 \delta x_2 + \dots K_n \delta x_n \quad , \tag{10.12}$$

which describes the relation between relative changes of different variables of the original equation (10.8) by a linear form.

See the logarithmic linearization of the model set-up above:

✈   In case of equation (10.1) the natural logarithm of both sides:

$$a = hc\alpha \quad \Rightarrow \quad \ln a = \ln(hc\alpha) = \ln h + \ln c + \ln \alpha \quad . \tag{10.13}$$

The total differential :

$$\frac{da}{a} = \frac{dh}{h} + \frac{dc}{c} + \frac{d\alpha}{\alpha} \quad , \tag{10.14}$$

then:

$$\delta a = \delta h + \delta c + \delta \alpha \quad . \tag{10.15}$$

✈   In case of equation (10.2) :

$$b = a + i + \beta \quad \Rightarrow \quad \ln b = \ln(a + i + \beta) \tag{10.16}$$

The total differential :

$$\frac{db}{b} = \frac{1}{a + i + \beta} da + \frac{1}{a + i + \beta} di + \frac{1}{a + i + \beta} d\beta \tag{10.17}$$

In this case every term should be multiplied by $\dfrac{x_i}{x_i}$:

$$\frac{db}{b} = \frac{a}{a(a + i + \beta)} da + \frac{i}{i(a + i + \beta)} di + \frac{\beta}{\beta(a + i + \beta)} d\beta \tag{10.18}$$

$$\delta b = \frac{a}{a + i + \beta} \delta a + \frac{i}{a + i + \beta} \delta i + \frac{\beta}{a + i + \beta} \delta\beta \tag{10.19}$$

✈   In case of equation (10.3):

$$c = a\gamma^\kappa \quad \Rightarrow \quad \ln c = \ln a + \kappa \ln \gamma \tag{10.20}$$

$$\frac{dc}{c} = \frac{da}{a} + \kappa \frac{d\gamma}{\gamma} \tag{10.21}$$

then:

$$\delta c = \delta a + \kappa \delta \gamma \quad . \tag{10.22}$$

✈   In case of equation (10.4):

$$e = \frac{2kl\omega^{\frac{\kappa-1}{\kappa}}}{b} \quad \Rightarrow \quad \ln e = \ln 2 + \ln k + \ln l + \frac{\kappa - 1}{\kappa} \ln \omega - \ln b \tag{10.23}$$

the derivative of a constant (*ln2*) equals zero, that is:

$$\frac{de}{e} = \frac{dk}{k} + \frac{dl}{l} + \frac{\kappa-1}{\kappa}\frac{d\omega}{\omega} - \frac{db}{b} \qquad (10.24)$$

in other form:

$$\delta e = \delta k + \delta l + \frac{\kappa-1}{\kappa}\delta\omega - \delta b \qquad (10.25)$$

The logarithmic linearization should be used in case of thermodynamic models and equations because of there are exponential terms.

10.3.2 Direct Differentiation

Using direct differentiation, as a first step, the total differential of both sides of the initial equation

$$y = f(x_1; x_2; \ldots x_n) \qquad , \qquad (10.26)$$

should be formed:

$$dy = \frac{\partial f(x_1; x_2; \ldots x_n)}{\partial x_1}dx_1 + \ldots + \frac{\partial f(x_1; x_2; \ldots x_n)}{\partial x_n}dx_n \qquad . \qquad (10.27)$$

Then both sides of the last equation should be multiplied by same sides of the general equation and all elements should be multiplied by $\dfrac{x_i}{x_i}$:

$$\frac{dy}{y} = \frac{\partial f(x_1; x_2; \ldots x_n)}{\partial x_1}\frac{x_1}{f(x_1; x_2; \ldots x_n)x_1}dx_1 + \ldots$$

$$\ldots + \frac{\partial f(x_1; x_2; \ldots x_n)}{\partial x_n}\frac{x_n}{f(x_1; x_2; \ldots x_n)x_n}dx_n \qquad (10.28)$$

Using the coefficients:

$$K_i = \frac{\partial f(x_1; x_2; \ldots x_n)}{\partial x_i}\frac{x_i}{f(x_1; x_2; \ldots x_n)} \qquad (10.29)$$

and equation (10.11), the following linear system can be achieved:

$$\delta y = K_1\delta x_1 + \ldots + K_n\delta x_n \qquad . \qquad (10.30)$$

This method is basically suggested if the general equation cannot be decomposed to multipliers.

10.3.3. The TAYLOR (LIE-MAGNUS) Series

In this case TAYLOR-series of the general equation

$$y = f(x_1; x_2 \ldots x_n) \qquad (10.31)$$

should be developed:

$$y + \Delta y = f(x_1; x_2 \ldots x_n) + \sum_{i=1}^{\infty}\frac{\partial^i f(x_1; x_2 \ldots x_n)}{\partial x_1^i}\frac{1}{i!}\Delta x_1^i + \ldots \qquad (10.32)$$

and its more than first-order terms have to be neglected

$$\Delta y = \frac{\partial f(x_1; x_2 \ldots x_n)}{\partial x_1} \Delta x_1 + \ldots \qquad . \qquad (10.33)$$

Then its both sides should be divided by the same side of the initial equation:

$$\frac{\Delta y}{y} = \frac{\partial f(x_1; x_2 \ldots x_n)}{\partial x_1} \frac{1}{f(x_1; x_2 \ldots x_n)} \frac{x_1}{x_1} \Delta x_1 + \ldots \quad . \qquad (10.34)$$

Using equations (10.11) and (10.29) the following equation can be achieved:

$$\delta y = K_1 \delta x_1 + \ldots + K_n \delta x_n \qquad . \qquad (10.35)$$

This linearization method can be used if the general equation can be derivable any times. The linearization using by TAYLOR series is applied basically in flight-mechanical investigations.

The LIE-MAGNUS series method is the so called matrix-form version of the TAYLOR series one that can be used for linearization of non-linear system of equations. This method uses the derivation matrix to derive the initial system of equations.

10.3.4. The Diagnostic Matrix

The equations (10.15) ; (10.19) ; (10.22) and (10.25) form a system of equations that is the linear (linearized) mathematical diagnostic model of system shown by Figure 10.1.

The linear system of equations achieved in this way describes interdependencies between relative changes of independent ($\delta \underline{x}$) and dependent ($\delta \underline{y}$) parameters from the point of view of the given investigation - see equations (10.6) and (10.7). This model can be written in the following matrix formula:

$$\underline{\underline{A}} \delta \underline{y} = \underline{\underline{B}} \delta \underline{x} \qquad , \qquad (10.36)$$

where $\underline{\underline{A}}$ and $\underline{\underline{B}}$ are coefficient matrices of external and internal parameters of the investigated system.

In case of system shown by Figure 10.1 the coefficient matrices are:

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ -\dfrac{a}{a+i+\beta} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \qquad , \qquad (10.37)$$

$$\underline{\underline{B}} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{i}{a+i+\beta} & \dfrac{\beta}{a+i+\beta} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \kappa & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & \dfrac{\kappa-1}{\kappa} \end{bmatrix} \qquad (10.38)$$

Using the

$$\underline{\underline{D}} = \underline{\underline{A}}^{-1}\underline{\underline{B}} \tag{10.39}$$

diagnostic matrix, the equation

$$\delta\underline{y} = \underline{\underline{D}}\delta\underline{x} \tag{10.40}$$

can be used for diagnostic investigations that will be shown in the following chapters.

10.4. Solutions of Non-Linear Mathematical Models

The non-linear mathematical model set-up in Chapter 10.2. should be solved. Because a real technical system is a multiple-unit one its mathematical model is a system of equations. In case of non-linear systems of equations few basic methods (for instance the chord-method) of solution of non-linear system cannot be used. In the technical practice the following methods are basically used to solve a non-linear system of equations:

✈ NEWTON-RAPSHON METHOD;
✈ GRADIENT METHOD;

10.4.1. The NEWTON-RAPSHON Method

To solve the non-linear system of equations

$$
\begin{aligned}
f_1(x_1;\ldots;x_n) &= 0 \\
&\vdots \\
f_n(x_1;\ldots;x_n) &= 0
\end{aligned}
\tag{10.41}
$$

suppose that x_{11} ; x_{21} … x_{n1} are its an approximate solution. In this case :

$$\Delta f_i = f_i(x_1;\ldots;x_n) \qquad . \tag{10.42}$$

Then the Taylor series of functions should be developed and its more then first-order terms have to be neglected:

$$
\begin{aligned}
\frac{\partial f_1}{\partial x_1}\Delta x_1 \quad \ldots \quad \frac{\partial f_1}{\partial x_n}\Delta x_n &= \Delta f_1 \\
&\vdots \\
\frac{\partial f_n}{\partial x_1}\Delta x_1 \quad \ldots \quad \frac{\partial f_n}{\partial x_n}\Delta x_n &= \Delta f_n
\end{aligned}
\tag{10.43}
$$

The linear system of equations (10.43) should be solved to vector $\Delta\underline{x}$ by any method and using this solution the next approximate solution is going to be:

$$
\begin{aligned}
x_{1,j+1} &= x_{1,j} + \Delta x_1 \\
&\vdots \\
x_{n,j+1} &= x_{n,j} + \Delta x_n
\end{aligned}
\tag{10.44}
$$

If all x_i fulfill the following inequality

$$\left| x_{i,j} \right| < \varepsilon \qquad , \tag{10.45}$$

the solution can be accepted, where:

ε - the acceptable deviation.

10.4.2. The Gradient Method

The essence of this method is that changing the value of the given scalar - vector function is studied in normal direction of the level surface in the *n*-dimensional space determined by the dependent variables of the function.

Starting from point $\underline{x}_0$ that means the zero-th approximate value belongs to level surface $f(\underline{x}) = f(\underline{x}_0)$, you should determine the normal direction that is the gradient of the surface at point $\underline{x}_0$. Along this gradient vector, you should determine the first approximate value $\underline{x}_1$ and its level surface $f(\underline{x}) = f(\underline{x}_1)$. Then you should determine point (and approximate value) $\underline{x}_2$ and its surface $f(\underline{x}) = f(\underline{x}_2)$ and so on …

Because

$$f(\underline{x}_0) > f(\underline{x}_1) > f(\underline{x}_2) > \dots \qquad , \tag{10.46}$$

you can get a point where the value of function $f(\underline{x})$ is the minimal. This vector is the solution of the given equation.
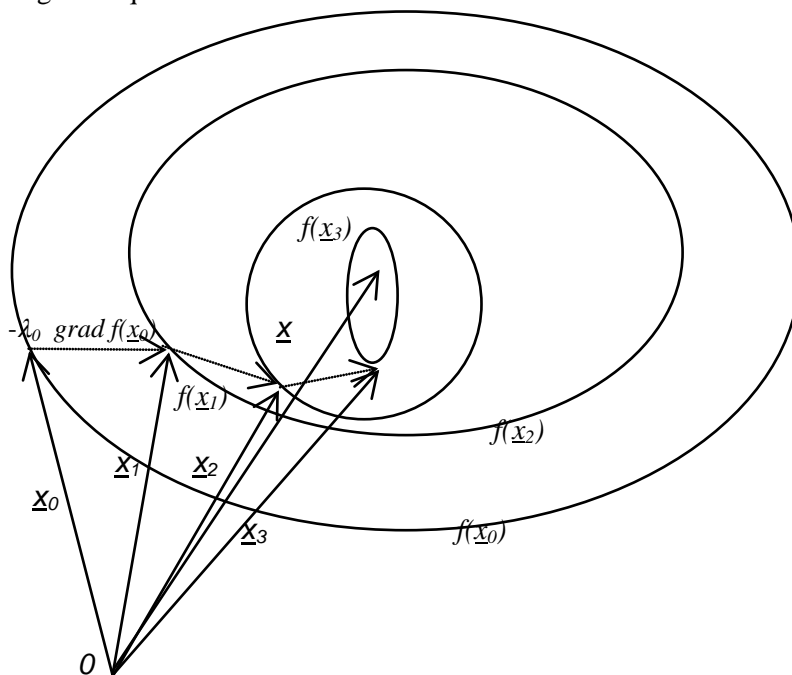
Figure 10.2 The Gradient Method

The gradient, which is the vector that shows the direction and intensity of the increasing of the scalar - vector function $f(\underline{x})$, can be determined by equation

$$grad f(\underline{x}) = \nabla f(\underline{x}) \qquad , \tag{10.47}$$

where:

∇ - Hamilton (nabla) operator:

$$\nabla = \begin{bmatrix} \dfrac{\partial}{\partial x_1} \\ \dfrac{\partial}{\partial x_2} \\ \vdots \\ \dfrac{\partial}{\partial x_n} \end{bmatrix} \tag{10.48}$$

The gradient method is illustrated by Figure 10.2 and its iteration equation is

$$\underline{x}_{i+1} = \underline{x}_i - \lambda_i \nabla f(\underline{x}_i) \qquad , \tag{10.49}$$

where:

λ_i -   multiplier of i-th iteration step

$i = 1 ; 2 ; ...$

SETTING UP OF MATHEMATICAL MODELS (EXAMPLES)

11.1. Introduction

Demonstrating the possibility of the use of the above-mentioned mathematical modeling methods, in this chapter the filling up of mathematical models will be shown in case of pneumatic system of the helicopter Mi-8 and the starter engine AI-9V.

Filling up of mathematical model of real technical system requires as to know exactly the given system, its work and its technical features. In this chapter the most minimal technical feature of the investigated systems and equipment will be shown because the author only would like to demonstrate the methodology and methods of mathematical diagnostic modeling.

11.2. The Pneumatic system of Helicopter Mi-8

The tasks of pneumatic systems of helicopter Mi-8 are to brake the main undercarriage wheels and to be compressed air-source in case of field operation. This system does not have anti block equipment and it cannot carry out differential braking for steering of helicopter during its taxing. Schematic diagram used in flying-technical practice of the system in shown by Figure 11.1. an the Table 11.1. shows its main technical data [1].

Tank pressure:	$40 - 50^{+4}$	bar
Volume of tanks	10 000	cm^3
Maximal control pressure:	11^{+1}	bar
Maximal brake pressure:	31^{+3}	bar

Table 11.1. Main Parameters of the System

11.2.1. Brake Control Valve PU-7

If the pilot pulls the brake lever being in the left control stick, it will have a wire and a lever moving the rod. It moves down and it has the reducing spring moving the piston down (see Figure 11.2) which is so to close the small releasing valve. The piston moves down too and it opens the small intake valve. The tank-pressure air streams out the space being under the big intake valve. The tank-pressure air

pushes down and opens the big intake valve that the compressed air streams into the space being under the piston and to the brake application valve UPO 3/2 . The pressure in under-piston - B - space increases and the compressed air pushes the piston up. The big intake valve will be closed slowly.
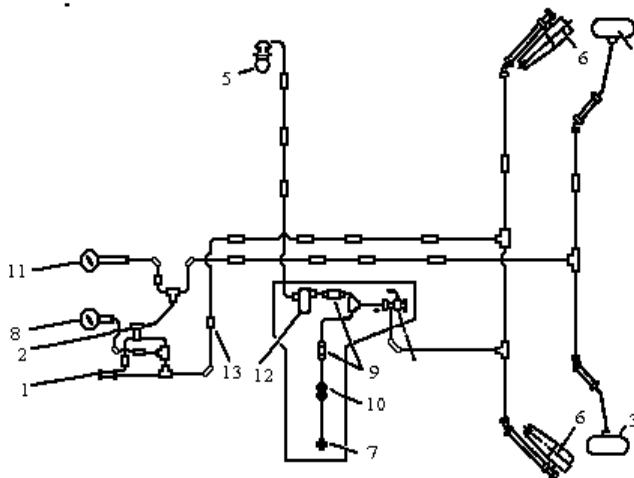

Figure 11.1. Pneumatic system of the helicopter Mi-8

1 - brake control valve PU-7; 2 - brake application valve UPO 3/2; 3 - brake-cylinders; 4 - automatic pressure controller AD-50; 5 - air compressor AK-50; 6 - tanks; 7 - ground connection; 8 - pressure-gage MV-60; 9 - one-way valve; 10;13 - air filter; 11 - pressure-gage MVU -100; 12 - sedimenter-filter;

If the forces acting upon the piston are in equilibrium, the piston will be stopped and control pressure (under the piston) will develop.

The forces which act upon the piston are the following:

downwards:

- force of the reducing spring F_{s1};

- ambient pressure $p_H A_p$;

upwards:

- control pressure $p_c A_p$;

- tank pressure $p_a A_s$;

- force of spring between small valves F_{s2};

Therefore the equilibrium:

$$p_c A_p + F_{s2} + p_a A_s = F_{s1} + p_H A_p \quad , \qquad (11.1)$$

and control-pressure (as the output parameter of PU-7):

$$p_c = \frac{F_{s1} + p_H A_p - p_a A_s - F_{s2}}{A_p} \quad \text{[Pa]} \qquad (11.2)$$

where:
A_p - area of brake control valve piston;
p_a - tank pressure after braking;
p_c - control pressure;
p_H - ambient pressure;
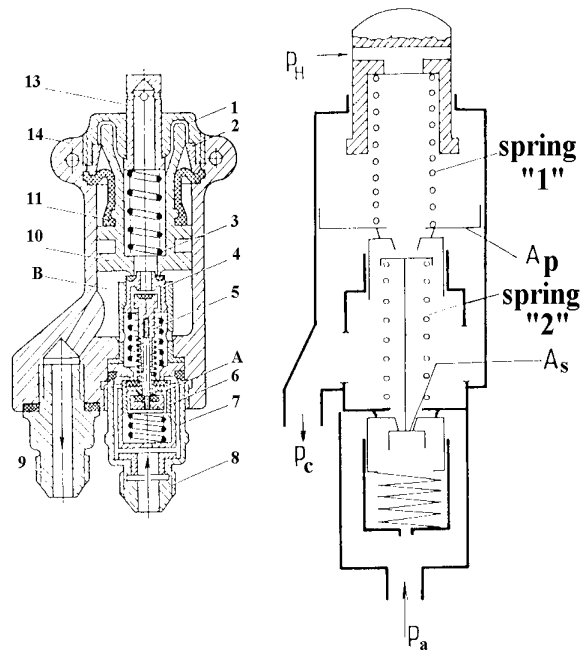
Figure 11.2. Brake Control Valve PU-7
A - tank-pressure chamber; B - control-pressure chamber.

11.2.2. The Brake Application Valve UPO 3/2

The control-pressure air pushes down the piston. It closes the bleading valve than opens the intake valve. Therefore tank-pressure air streams into the space being under the piston and to the brake-cylinders. If the pressure of under-piston space increases, the piston will move up and intake valve will be closed (Figure 11.3).

If the forces which act upon the piston are in the equiblirium, the piston will stand and the valves will be closed. In this case brake-pressure is developed and the system brakes the main undercarriage wheels of the helicopter.

The forces which act upon the piston are the following:
downwards:

   - control pressure $\qquad\qquad p_c A_1$;

upwards:

   - brake pressure $\qquad\qquad p_b A_3$;

   - ambient pressure $\qquad\qquad p_H A_2$;

   - force of the spring $\qquad\qquad F_{s3}$.

$$p_c A_1 = p_H A_2 + F_{s3} + p_b A_3 \tag{11.3}$$

Then the brake pressure:

$$p_b = \frac{p_c A_1 - p_H A_2 - F_{s3}}{A_3} \quad [\text{Pa}] \tag{11.4}$$

where:

A_n - area of n-th piston of brake application valve;

p_b - pressure of brake-air;
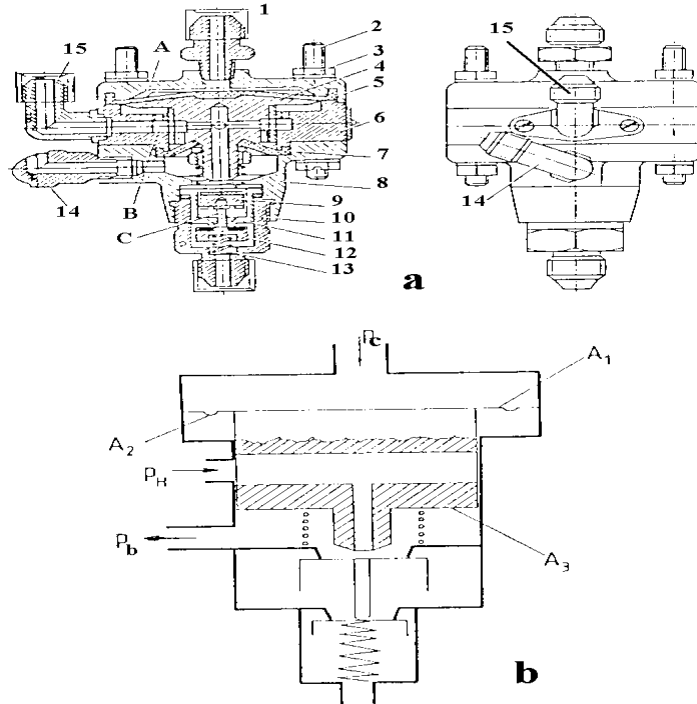
Figure 11.3. The Brake Application Valve UPO 3/2

A - control-pressure chamber; B - ambient-pressure chamber; C - tank-pressure chamber.

11.2.3. The Brake Assembles

They have two brake-blocks and two cylinders. Their brake clearance can be adjusted.

The brake force depends on the followings:

- friction coefficient μ_j;

- prestressing force of the restoring spring F_{rj};

$$F_{rj} = x_0 s_j \tag{11.5}$$

- brake-shoe clearance z_j;

- stiffness of the restoring spring s_j;
- difference between the brake and ambient pressures $p_b - p_H$;
{ - "piston - brake-shoe" transmission i_{pj} };
{ - "restoring spring - brake-shoe" transmission i_{sj} }.
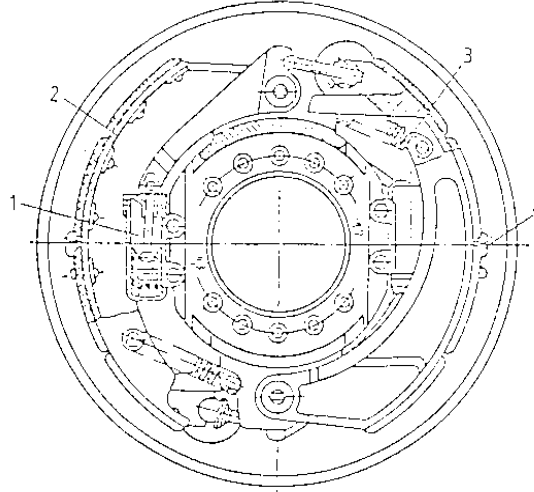
Figure 11.4. The Brake Assemble

$$F_j = \left[(p_b - p_H)A_c i_{pj} - \left(\frac{i_{pj}}{i_{sj}} \right)^2 z_j s_j - \frac{i_{pj}}{i_{sj}} x_0 s_j \right] \mu_j \qquad (11.6)$$

that is:

$$F_j = \left[(p_b - p_H)A_c i_{pj} - \frac{i_{pj}}{i_{sj}} \left(\frac{i_{pj}}{i_{sj}} z_j + x_0 \right) s_j \right] \mu_j \quad [\text{N}] \qquad (11.7)$$

where:

A_c - area of brake-cylinder;

x_0 - prestressing distance;

11.2.4. The Tanks

The helicopter Mi-8 has two compressed-air tanks. Their task is the compressed air conservation. Both of them have 5000 cm^3 volume. They are in the stiffening beam of the main undercarriages.

During the braking compressed air is led up to

- control-pressure sub-system
(PU-7, UPO 3/2 equipment and connecting tube) $V_{cp}(p_c - p_H)$

-   brake-pressure sub-system;

(UPO 3/2, brake cilinders and connecting tube) $\left(V_{pl} + \sum_{j=1}^{4} i_{pj} z_j A_{bj} \right)(p_b - p_{H)}$

Supposing adiabatic expansion the tank pressure after braking is:

$$p_a = \left[p_t - \frac{V_{cp}(p_c - p_H) + \left(V_{pl} + \sum_{j=1}^{4} i_{pj} z_j A_{bj} \right)(p_b - p_{H)})}{V_t} \right]$$

$$\left[1 - \frac{V_{cp}(p_c - p_H) + \left(V_{pl} + \sum_{j=1}^{4} i_{pj} z_j A_{bj} \right)(p_b - p_{H)})}{V_t p_t} \right]^{\kappa - 1} \quad \text{[Pa]}$$

(11.8)

where:
p_t - tank pressure before braking;
V_{cp}- volume of the control-pressure sub-system;
V_{pl}- volume of the pipe-line;
V_t - volume of the tank;

11.2.5. Linearization

After their logarithmic linearization:

the equation (11.2) became
$$\delta p_c = K_1 \delta F_{s1} - K_2 \delta F_{s2} - K_3 \delta p_a + K_4 \delta p_H \quad ; \tag{11.9}$$
the equation (11.4) became
$$\delta p_b = K_5 \delta p_c - K_6 \delta F_{s3} - K_7 \delta p_H \quad ; \tag{11.10}$$
the equation (11.7) became
$$\delta F_j = K_8 \delta p_b - K_9 \delta p_H - K_{10} \delta z_j - K_{11} \delta s_j \quad ; \tag{11.11}$$
the equation (11.8) became
$$\delta p_a = K_{12} \delta p_t - K_{13} \delta p_c - K_{14} \delta p_b + K_{15} \delta p_H + \sum_{j=1}^{4} K_{16} \delta z_j \quad . \tag{11.12}$$

For the following diagnostic investigations, independent parameters are the input ones of the units of the system. The dependent parameters ($\delta \underline{y}$) and independent ones ($\delta \underline{x}$) should be separated into the following vectors:

$$\delta \underline{y}^T = \begin{bmatrix} \delta p_c & \delta p_b & \delta F_1 & \delta F_2 & \delta F_3 & \delta F_4 & \delta p_a \end{bmatrix} \tag{11.13}$$

$$\delta \underline{x}^T = \begin{bmatrix} \delta F_{s1} & \delta F_{s2} & \delta p_H & \delta F_{s3} & \delta z_1 & \delta s_1 & \delta z_2 & \delta s_2 & \delta z_3 & \delta s_3 & \delta z_4 & \delta s_4 & \delta p_t \end{bmatrix}$$

$$\tag{11.14}$$

$$K_1 = \frac{F_{s1}}{C_1} \quad ; \qquad K_2 = \frac{F_{s2}}{C_1} \quad ; \qquad K_3 = \frac{A_s p_a}{C_1} \quad ; \qquad K_4 = \frac{A_p p_H}{C_1}$$

$$C_1 = F_{s1} - F_{s2} + A_p p_H - A_s p_a$$

$$K_5 = \frac{p_c A_1}{C_2} \quad ; \qquad K_6 = \frac{F_{s3}}{C_2} \quad ; \qquad K_7 = \frac{A_2 p_H}{C_2}$$

$$C_2 = p_c A_1 - F_{s3} - p_H A_2$$

$$K_8 = \frac{A_{cj} i_{pj} p_b}{C_3} \quad ; \qquad K_9 = \frac{A_{cj} i_{pj} p_H}{C_3} \quad ;$$

$$K_{10} = \frac{\left(\dfrac{i_{pj}}{i_{sj}}\right)^2 s_j z_j}{C_3} ; \qquad K_{11} = \frac{\dfrac{i_{pj}}{i_{sj}}\left(\dfrac{i_{pj}}{i_{sj}} z_j + x_0\right) s_j}{C_3} \quad ;$$

$$C_3 = (p_b - p_H) A_{cj} i_{pj} - \frac{i_{pj}}{i_{sj}}\left(\frac{i_{pj}}{i_{sj}} z_j + x_0\right) s_j$$

$$K_{12} = \frac{p_t}{C_4} \quad ; \qquad K_{13} = \frac{V_{cp} p_c}{V_t C_4} ; \qquad K_{14} = \frac{\left(V_{pl} + \sum_{j=1}^{4} i_{pj} z_j A_{bj}\right) p_b}{C_4} \quad ;$$

$$K_{15} = \frac{\left(V_{cp} + V_{pl} + \sum_{j=1}^{4} i_{pj} z_j A_{bj}\right) p_H}{C_4} \quad ; \qquad K_{16} = \frac{i_{pj} z_j A_{bj}(p_b - p_H)}{C_4}$$

$$C_4 = p_t - \frac{V_{cp}(p_c - p_H) + \left(V_{pl} + \sum_{j=1}^{4} i_{pj} z_j A_{bj}\right)(p_b - p_H)}{V_t p_t}$$

Table 11.2. The Elements of the Coefficient Matrices

The coefficient matrices of the dependent and independent parameters are

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & K_3 \\ -K_5 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -K_8 & 1 & 0 & 0 & 0 & 0 \\ 0 & -K_8 & 0 & 1 & 0 & 0 & 0 \\ 0 & -K_8 & 0 & 0 & 1 & 0 & 0 \\ 0 & -K_8 & 0 & 0 & 0 & 1 & 0 \\ K_{13} & K_{14} & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{11.15}$$

$$\underline{\underline{B}}^T = \begin{bmatrix} K_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -K_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ K_4 & -K_7 & -K_9 & -K_9 & -K_9 & -K_9 & K_{15} \\ 0 & -K_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -K_{10} & 0 & 0 & 0 & K_{16} \\ 0 & 0 & -K_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -K_{10} & 0 & 0 & K_{16} \\ 0 & 0 & 0 & -K_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -K_{10} & 0 & K_{16} \\ 0 & 0 & 0 & 0 & -K_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -K_{10} & K_{16} \\ 0 & 0 & 0 & 0 & 0 & -K_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -K_{12} \end{bmatrix} \tag{11.16}$$

The elements of the coefficient matrices - multipliers of the equations (11.9) - (11.12)- are determined by the Table 11.2.

11.3. The Starter Engine AI-9V

The tasks of starter engine Ai-9V are to assure compressed air to starter-turbines of main engines of the helicopter to start them („air-topping" regime) and to supply the direct current electrical system of the helicopter as an emergency power source („generator" regime). The Figures 11.5. and 11.6. show the characteristics of the compressor and turbine of the engine, which will be necessary for mathematical modeling.

For exact and accurate determination of momentary regime of this engine four parameter have to be known. Therefore its mathematical model have to be a system of equation that was four equation. They are equations which depict - in case of the air-topping regime:

✈   the equality of mass flow
between compressor and turbine:

$$\dot{m}_k - \dot{m}_1 + \dot{m}_{fl} - \dot{m}_T = 0 \quad , \tag{11.17}$$

where:

$\dot{m}_k$ -   mass flow of the compressor;

$\dot{m}_1$ -   mass flow to main engines;

$\dot{m}_{fl}$ -   fuel mass flow;

$\dot{m}_T$ -   mass flow of the turbine.

between turbine and exhaust nozzle:

$$\dot{m}_T - \dot{m}_{en} = 0 \quad , \tag{11.18}$$

where:

$\dot{m}_{en}$ -   mass flow of the exhaust nozzle.

✈   the equality of the powers

between compressor and turbine:

$$P_t \eta_m - P_k - P_a = 0 \quad , \tag{11.19}$$

where:

P_t -   power of the turbine

η_m -   mechanical efficiency of the rotor;

P_k -   required power of the compressor;

P_a -   power of the auxiliary equipment.

✈   control law:

$$n = const. \quad , \tag{11.20}$$

where:

n -   momentary number of revaluation;

   The equations (11.17) - (11.19) form the thermodynamic mathematical model of starter engine AI-9V in case of air-topping regime. To solve them, their inner relations should be discovered. Since inner relations mentioned above the mathematical model will be a non-linear one.
   In the following, the inner relations of the thermodynamic model will be shown shortly and basically.

11.3.1. The Ambient Parameters

The ambient parameters should be determined on the basis of International Standard Atmosphere [24]:

✈   ambient (inlet) temperature:
$$T_1(H) = 288 - 0{,}0065H \qquad \text{[K]} \quad ;$$
(11.21)

where:
H -   flight level [m]

✈   ambient (inlet) pressure:
$$p_1(H) = 101325\left(1 - \frac{H}{44300}\right)^{5{,}256} \qquad \text{[Pa]} \quad ;$$
(11.22)

11.3.2. The Compressor

✈   Temperature after compressor:
$$T_2 = T_1\left(1 + \frac{\pi_k^{\frac{\kappa-1}{\kappa}}}{\eta_k}\right) \qquad \text{[K]} \quad ;$$
(11.23)

where:
κ -   adiabatic exponent [–];
π_k -   pressure-ratio of the compressor [–];
η_k -   isentropic efficiency of the compressor [–].

The latter two parameters should be determined by characteristic of the compressor shown by Figure 11.5.

✈   mass flow of the compressor:
$$\dot{m}_k = \left(\frac{2}{\kappa+1}\right)^{\frac{1}{\kappa-1}}\sqrt{\frac{2}{(\kappa+1)R}}\,\frac{\sigma_{bcs}p_1}{\sqrt{T_1}}\,A_k q(\lambda)_k \quad \text{[kgs}^{-1}\text{]};$$
(11.24)

where:
A_k -   area of inlet scoop [m^2];
σ_{bcs} -   coefficient of pressure loss of inlet scoop [–];
R -   specific gas constant [kJ(kgK)$^{-1}$];
$q(\lambda)_k$ -   non-dimensional mass flow rate of the compressor [–].

✈   power of the compressor:
$$P_k = c_{pl}(T_2 - T_1)\dot{m}_k \qquad \text{[kW]} \quad ;$$
(11.26)

where:

c_{pl} -   mass specific heat at constant pressure of air $[kJ(kgK)^{-1}]$.
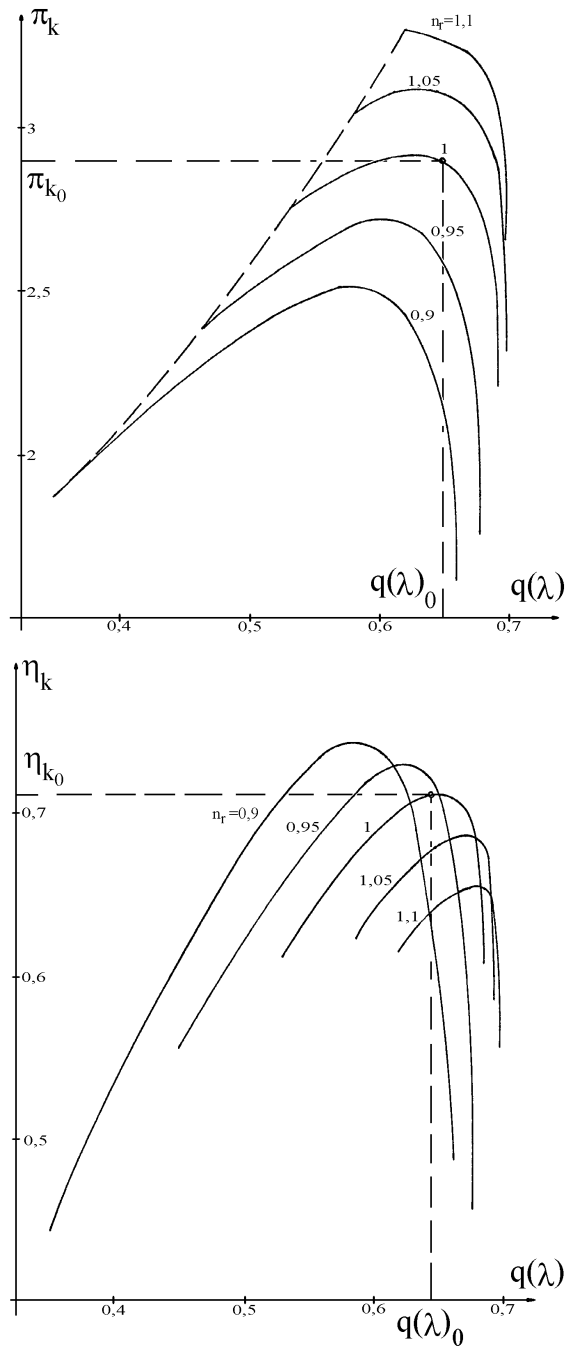

Figure 11.5. Characteristic of the Compressor

✈   pressure after compressor:

$$p_2 = \pi_k \sigma_{bcs} p_1 \qquad \text{[Pa]} \qquad . \qquad (11.27)$$

11.3.3 The Combustion Chamber

The gas temperature after combustion chamber can be determined using method and equation developed by Professor SÁNTA [28, 29]:

$$T_3 = \frac{-bj + \sqrt{bj^2 - 4acbi}}{2ac} \qquad \text{[K]} \quad ; \qquad (11.28)$$

where:

$bj = 1,07626 + 1,823 q_t$

$bi = -71,972 - 168,206 q_t - bk$

$ac = 4,05714 \cdot 10^{-5} + 6,568 \cdot 10^{-4} q_t$

$bk = \eta_{et} q_t + c_p (1 + q_t)$

$c_p = 18,107 + 0,912577 T_2 + 1,151786 T_2^2$

q_t -   fuel - air mass flow ration [–];

η_{et} -   efficiency of the combustion chamber [–].

✈   pressure after combustion chamber

$$p_3 = p_2 \sigma_{et} \qquad \text{[Pa]} \quad . \qquad (11.29)$$

11.3.4. The Turbine

✈   Temperature after the turbine:

$$T_4 = T_3 \left(1 - \eta_T \left\{ 1 - \frac{1}{\pi_T^{\frac{\kappa-1}{\kappa}}} \right\} \right) \qquad \text{[K]} \quad ; \qquad (11.30)$$

where:

η_T -   isentropic efficiency of the turbine [–];

π_T -   pressure ratio of the turbine [–].

The efficiency of the turbine should be determined by characteristics of the turbine - see Figure 11.6.

✈   mass flow of the turbine:

$$\dot{m}_T = \left(\frac{2}{\kappa+1} \right)^{\frac{1}{\kappa-1}} \sqrt{\frac{2}{(\kappa+1)R}} \frac{p_3}{\sqrt{T_3}} A_T q(\lambda)_T \qquad \text{[kgs}^{-1}\text{]} \qquad (11.31)$$

where:

A_T -   minimal area of row of the blades of the turbine $[m^2]$;

$q(\lambda)_T$ -   non-dimensional mass flow rate of the turbine $[-]$.

The non-dimensional mass flow rate of the turbine can be determined on the basis of the characteristic of the turbine, which can be seen in Figure 11.6.

✈   power of the turbine:

$$P_t = \overset{\bullet}{m}_T \, c_{pg} (T_3 - T_4) \qquad [W] \quad , \qquad (11.32)$$

where:

c_{pg} -   mass specific heat at constant pressure of the gas $[kJ(kgK)^{-1}]$.
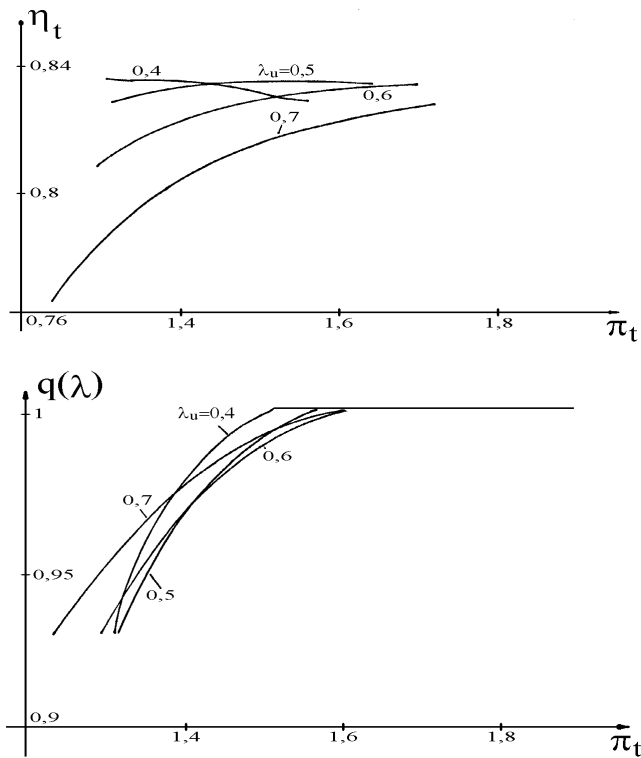
Figure 11.6. Characteristic of the Turbine

✈   pressure after the turbine:

$$p_4 = \frac{p_3}{\pi_T} \qquad [Pa] \quad . \qquad (11.33)$$

11.3.5. The Exhaust Nozzle

✈   The pressure ratio of the exhaust nozzle:

$$\pi_{en} = \frac{p_4}{p(H)} \qquad [-] \qquad\qquad (11.34)$$

✈   non-dimensional velocity of exhaust nozzle at the exit:

$$\lambda_{en} = \sqrt{\left(1 - \frac{1}{\pi_{en}^{\frac{\kappa-1}{\kappa}}}\right)\frac{\kappa+1}{\kappa-1}} \quad if \quad \pi_{en} < \left(\frac{\kappa+1}{2}\right)^{\frac{\kappa}{\kappa-1}}$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad [-] \qquad (11.35)$$

$$\lambda_{en} = 1 \qquad\qquad\qquad if \quad \pi_{en} \geq \left(\frac{\kappa+1}{2}\right)^{\frac{\kappa}{\kappa-1}}$$

✈   non-dimensional mass flow rate of the exhaust nozzle:

$$q(\lambda)_{en} = \lambda_{en}\left(1 - \frac{\kappa-1}{\kappa+1}\lambda_{en}^2\right)^{\frac{1}{\kappa-1}}\left(\frac{\kappa+1}{2}\right)^{\frac{1}{\kappa-1}} \qquad [-] \qquad (11.36)$$

✈   mass flow of the exhaust nozzle:

$$\dot{m}_{en} = \left(\frac{2}{\kappa+1}\right)^{\frac{1}{\kappa-1}}\sqrt{\frac{2}{(\kappa+1)R}}\frac{p(H)}{\sqrt{T_4}}A_{en}q(\lambda)_{en} \qquad [kgs^{-1}] \qquad (11.37)$$

APPLICATION OF THE MODELS FOR DIAGNOSTIC INVESTIGATION

12.1 Introduction

In this chapter the diagnostic investigations which are based on mathematical modeling will be shown. This chapter will show their methodology and then illustrate their possibilities of use by the examples of pneumatic brake-system of the helicopter Mi - 8 *Hip* and starter engine AI - 9V.

12.2 The Sensitivity Test

12.2.1 Methodology

The essence of the sensitivity test is that the failure or operational wear-out of the given functional unit is simulated by changing of its independent variables (only in the diagnostic model and not in the real system!) [29]. On the basis the linear or non-linear mathematical model of the investigated system, it can be determined how sensitive the dependent system variables will be to the simulated changes.

If only one of the independent variables is changed, the investigation will be called an one-parameter sensitivity test. If the number of the changed independent variables is more than one, the several-parameter sensitivity test is used.

It is important to mention that the changes of independent variables cannot be more than about 1 or 2 % depending on the intensity of the original model's non-linearity. These models are basically either linearized one or the non-linear one which need to be solved by any numerical methods, which usually bases on linearization. Therefore, depending on the non-linearity of the original model, the results of the sensitivity test can have difference from real influences of the simulated changes. But the result shows the direction and order of magnitude of the real influences with enough accuracy

The results of sensitivity test can be used for the conclusions to come about the technical features of the given system and its behavior in case of simulated failures without that you put out of order the real system during the planning or modification of the system.

This method can be used for troubleshooting too. In case of malfunction of the system for localization of the trouble, we can test the fault supposed logically by us.

12.2.2 Case of Pneumatic System of Helicopter Mi - 8

In this case we use the linear mathematical diagnostic model of the pneumatic brake-system of the helicopter Mi - 8 - see equations (11.13) …(11.16) [12;21].For the one-parameter sensitivity test the following independent parameters were changed:

✈   tank pressure prior to braking - p_t;
✈   force of the reducing spring of the PU-7 - F_{s1};
✈   stiffness of restoring spring of the 2nd brake-shoe - s_2;
✈   clearance of the 2nd brake-shoe - z_2.

p_c	p_b	F_1	F_2	F_3	F_4	p_a
$\Delta p_t = -1\%$ (0,5 bar)						
3,492e-1	3,764e-1	3,960e-1	3,960e-1	3,960e-1	3,960e-1	-1,022
$\Delta F_{r1} = -1\%$ (6,1 N)						
1,273	-1,373	-1,444	-1,444	-1,444	-1,444	2,279e-2
$\Delta s_2 = -1\%$ (4,53 Nmm^{-1})						
0	0	0	1,79e-2	0	0	0
$\Delta z_2 = -1\%$ (4 µm)						
1,416e-5	1,527e-5	1,606e-5	1,285e-3	1,606e-5	1,606e-5	-4,147e-5
$\Delta z_2 = -1\%$ and $\Delta s_2 = -1\%$						
1,416e-5	1,527e-5	1,606e-5	1,9205e-2	1,606e-5	1,606e-5	-4,147e-5

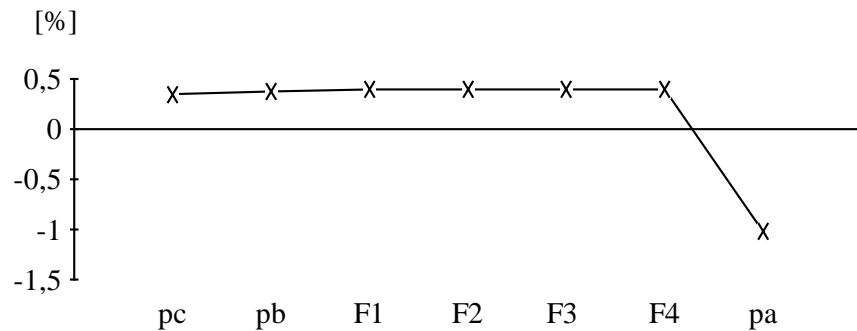Table 12.1. Results of Sensitivity Test

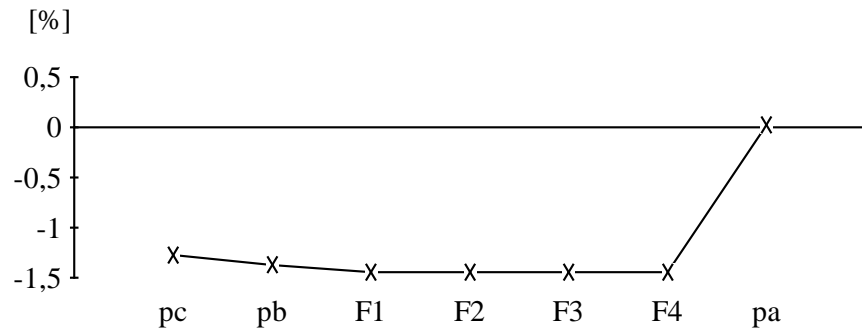Figure 12.1 Sensitivity of the System ($\Delta p_t = -1\%$)

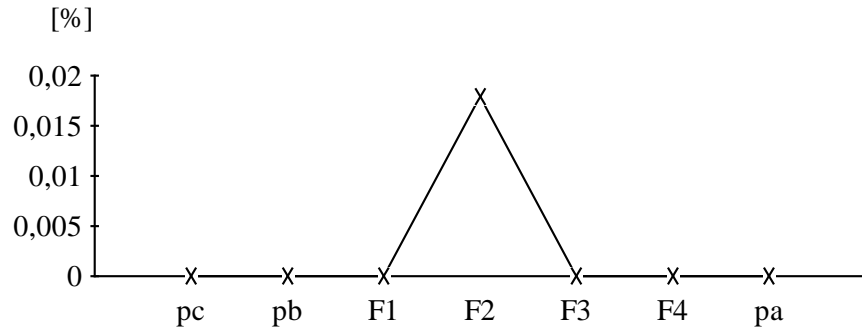Figure 12.2.   Sensitivity of the System ($\Delta F_{r1} = -1\%$)

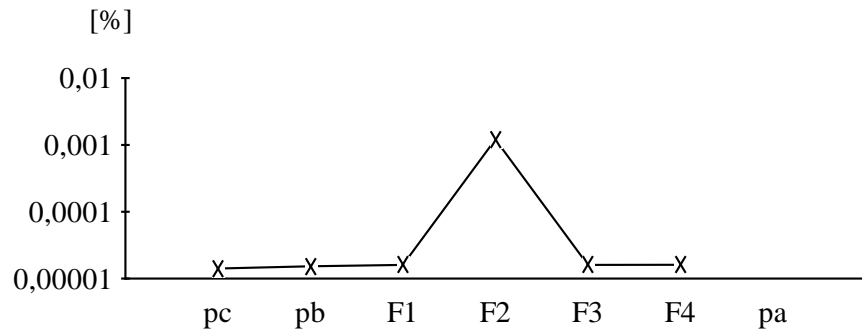Figure 12.3.   Sensitivity of the System ($\Delta s_2 = -1\%$)

Figure 12.4.   Sensitivity of the System ($\Delta z_2 = -1\%$)

For two-parameter sensitivity-test of the system, as an example, the 1 % decreasing of the stiffness of restoring spring and clearance of the 2nd brake-shoe has been simulated. Its result can be seen in the figure 12.5.
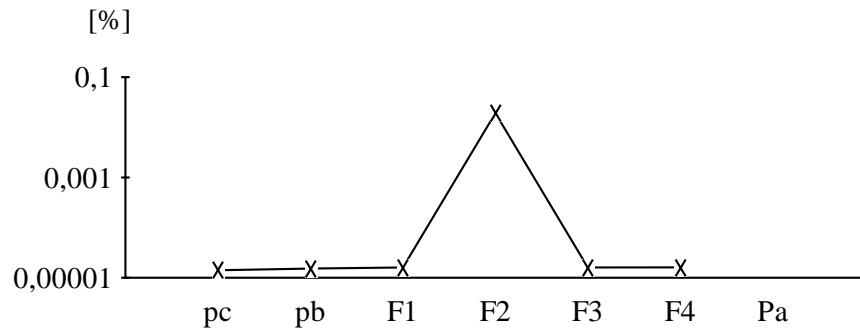
[%]

Figure 12.5.   Two-Parameter Sensitivity of the System ($\Delta z_2 = -1\%$ and $\Delta s_2 = -1\%$)
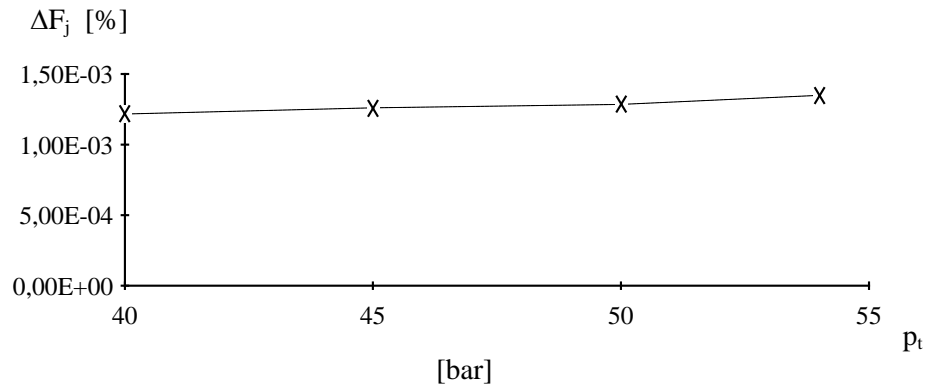

Figure 12.6 Sensitivity of the System Depending on the Tank Pressure Prior to Braking ($\Delta z_j = -1\%$)
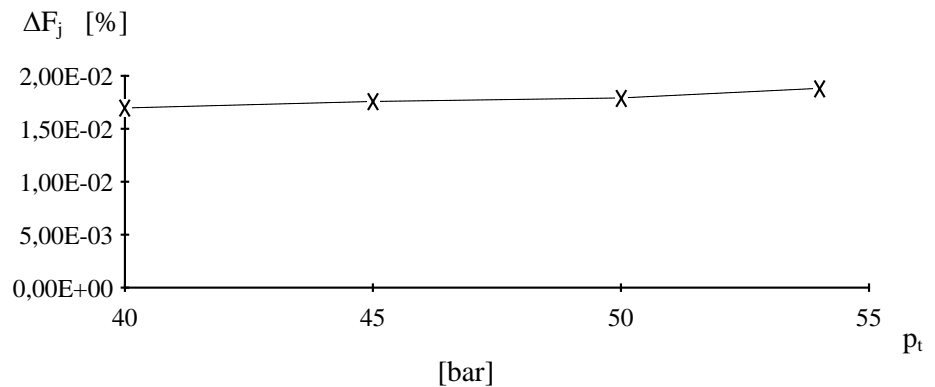

Figure 12.7. Sensitivity of the System Depending on the Tank Pressure Prior to Braking ($\Delta s_j = -1\%$)

   The sensitivity test can be used in different match-points of the system. Then the elements of the coefficient matrices have to be determined for all match-points. For demonstration, the sensitivity-test have been performed in cases of different tank-pressure prior to braking p_t. You can see the influences of the 1 % decreases

of clearance of the *j*-th brake-shoe z_j (figure 12.6) and its stiffness of the restoring spring (figure 12.7) depending on the tank pressure prior to braking p_t . Figure 12.8 and 12.9 show the change of brake force of the *j*-th brake-shoe in cases of decrease of its clearance and of stiffness of its restoring spring depending on the brake pressure p_b .
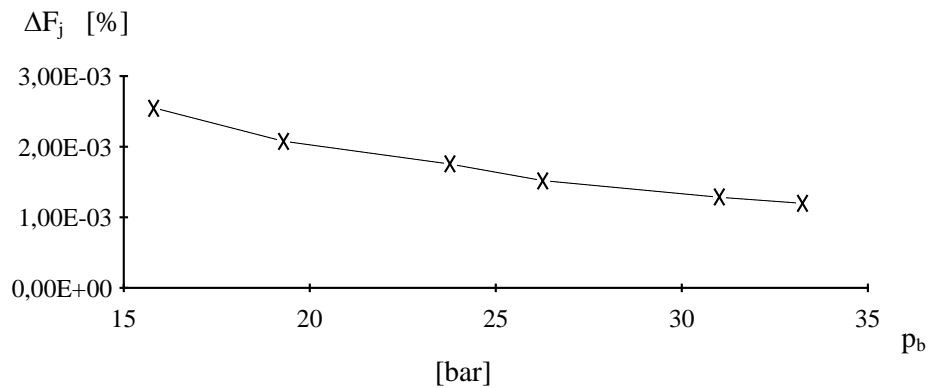

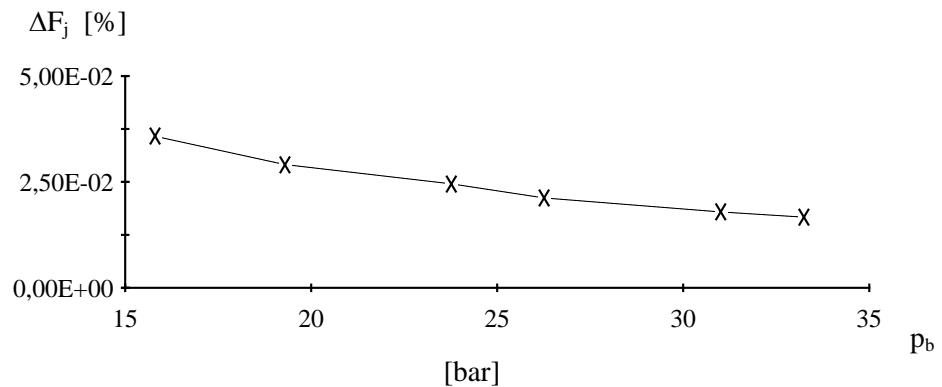Figure 12.8. Sensitivity of the System Depending on the Brake Pressure (Δz_j = - 1%)

Figure 12.9. Sensitivity of the System Depending on the Brake Pressure (Δs_j = - 1%)

On the basis of this test you can state it that:

✈   the examined system is very insensitive to the anomalies of the operational parameters;

✈   the sensitivity of the system decreases if the brake pressure is increased to its nominal maximum value.

These can be considered beneficial with respect to the operation since great deviations in the internal parameters are admissible in the course of operation. For example, longer period between the checkings, repairs and overhauls of maintenance can be determined.

12.2.2 Case of the Starter Engine AI - 9V

For one-parameter sensitivity test of starter engine AI - 9 V, its non-linear stationary thermodynamical model - see equations (11.17) … (11.20) - is used. The following independent parameters are changed:

→ isentropic efficiency of the turbine - η_t ;
→ isentropic efficiency of the compressor - η_k ;
→ mechanical efficiency - η_m ;
→ area of inner scoop - A_k ;
→ stagnation pressure recovery coefficient of the combustion chamber - σ_{et} ;
→ stagnation pressure recovery coefficient of the inner duct - σ_{bcs} ;
→ efficiency of combustion chamber - η_{et};

The result of the above investigation can be seen in the Table 12.2 and in the Figures 12.10 - 12.16.

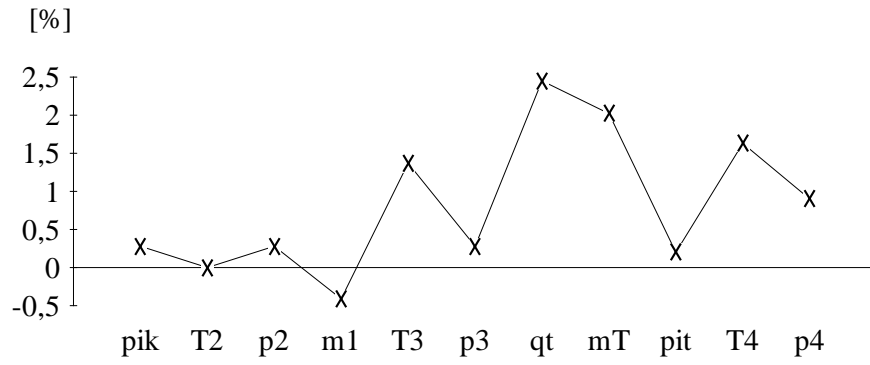| π_k	T_2	p_2	m_1	T_3	p_3	q_t	m_t	π_t	T_4	p_4
$\delta\eta_t = -1$ %										
0,28		0,28	-0,41	1,37	0,28	2,45	2,02	0,2	1,63	0,09
$\delta\eta_k = -1$ %										
0,03		0,03	-0,05	0,17	0,03	0,26	0,21	0,03	0,2	0,01
$\delta\eta_m = -1$ %										
0,23		0,23	-2,59	1,12	0,23	1,99	1,65	0,18	1,12	0,05
$\delta A_k = -1$ %										
-0,50	-0,23	-0,50	-0,36	-0,26	-0,50	-0,42	-0,78	-0,40	-0,20	-0,10
$\delta\sigma_{et} = -1$ %										
0,57	0,23	0,57	-0,88	0,77	-0,5	1,28	0,39	-0,4	0,82	-0,1
$\delta\sigma_{bcs} = -1$ %										
0,09		-0,92	1,13	0,43	-0,92	0,74	-0,41	-0,74	0,51	-0,18
$\delta\eta_{et} = -1$ %										
						1,01	1,1			

Table 12.2. Results of Sensitivity Test

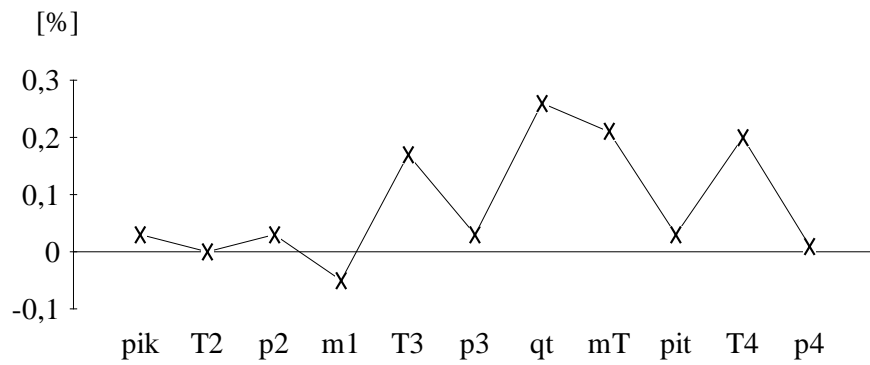Figure 12.10. Sensitivity of the Starter Engine $\delta\eta_t = -1$ %

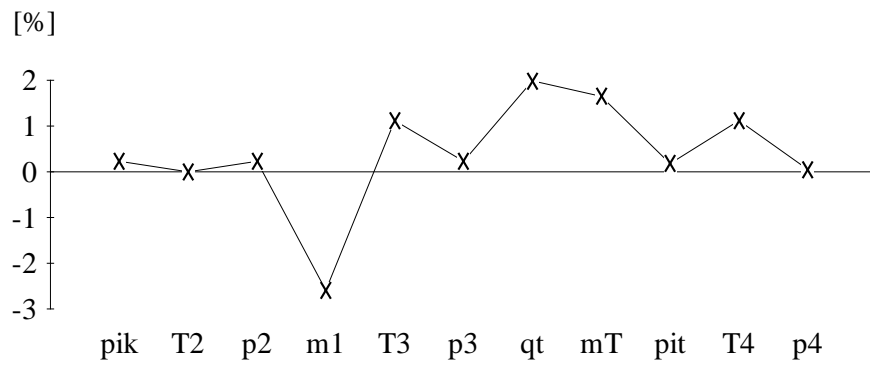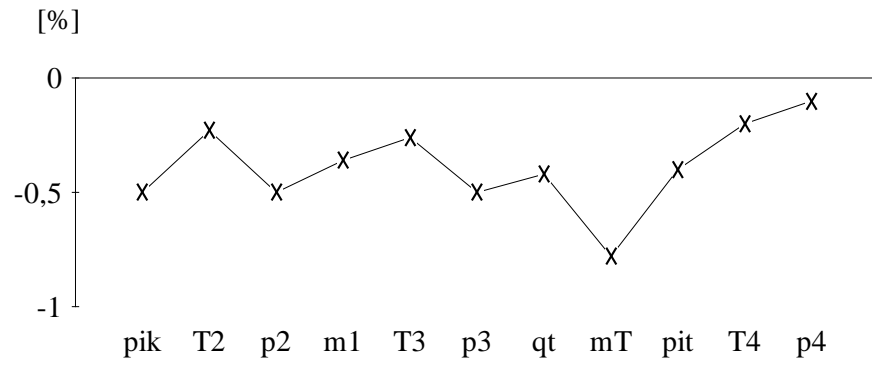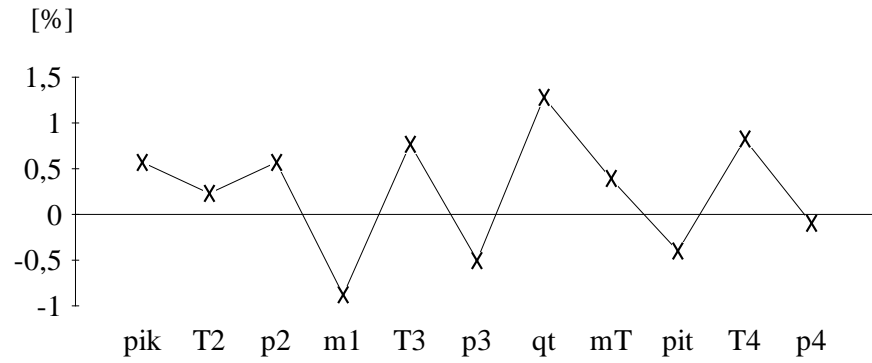Figure 12.11. Sensitivity of the Starter Engine $\delta\eta_k = -1$ %
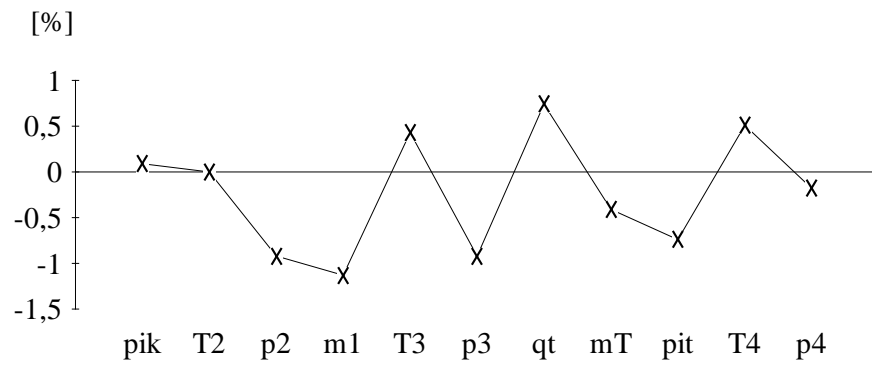

Figure 12.12. Sensitivity of the Starter Engine $\delta\eta_m = -1$ %

[%]

Figure 12.13. Sensitivity of the Starter Engine δA_k = -1 %

[%]

Figure 12.14. Sensitivity of the Starter Engine $\delta \sigma_{et}$ = -1 %

[%]

Figure 12.15. Sensitivity of the Starter Engine $\delta \sigma_{bcs}$ = -1 %
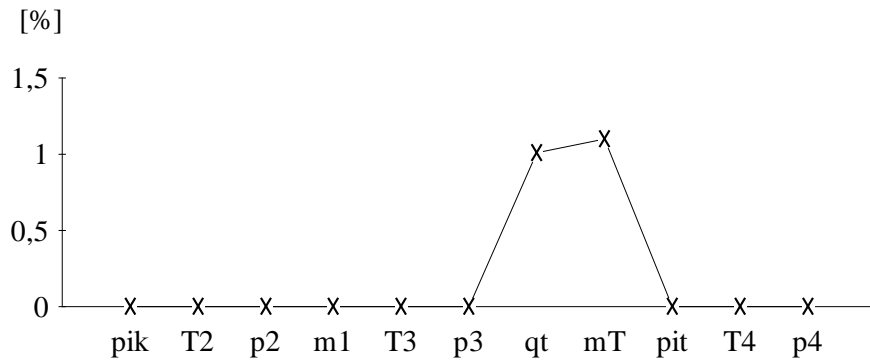
Figure 12.16. Sensitivity of the Starter Engine $\delta\eta_{et} = -1$ %

The conclusions of the above investigation are the followings::

✈   the mass flow rate to main engines m_1 decreases in all simulated cases;
✈   the absolute temperature after T_2 the compressor changes the most minimally;
✈   the parameters of the inlet scoop (A_{bcs} , σ_{bcs}) have the most minimal influences an the cycle of the engine that is on the investigated parameters;
✈   the efficiency of the combustion chamber η_{et} has only the fuel mass parameters (q_t and m_t) increased;
✈   the mechanical efficiency η_m has the most influences on the above-investigated external parameters.

12.2.4. Determination of Diagnostic Matrix

There is an other method to determine the diagnostic matrix of examined system that is based on usage of its non-linear mathematical model.

If the original non-linear mathematical model has very sophisticated inlet relationships and so functions (for example in case of gas-turbine engines), the linearization by any method is very difficult and complicated. In this case the following method should be used.

The investigated external and internal parameters should be chosen and arranged into vectors $\underline{x}$ and $\underline{y}$. The non-linear model should be excited by the 1 % perturbations of the above-chosen external parameters one by one.

Knowing the result of base and "perturbed" models, the relative changes of chosen internal parameters should be determined by the equation

$$\delta\eta = \frac{\eta_{pert.} - \eta_{base}}{\eta_{base}} \quad , \tag{12.1}$$

where:

$\eta_{pert.}$ -   "perturbed" value of general parameter η ;

η_{base} -   base value of general parameter η .

These relative changes should be arranged into the diagnostic matrix $\underline{\underline{D}}$ depending on the above-determined vectors $\underline{x}$ and $\underline{y}$.

To demonstrate the possibility of to use of this method, it was used in case of non-linear mathematical model of starter-engine AI - 9V by a very simple example. For investigation, the following external

➤   pressure ratio of compressor - π_k;
➤   increase of temperature in the compressor - ΔT_k;
➤   fuel mass flow ratio - $\dot{m}_t$;
➤   absolute temperature after the turbine - T_4;
➤   mass flow ratio to main engines - $\dot{m}_1$,

$$\underline{y}^T = \begin{bmatrix} \pi_k & \Delta T_k & \dot{m}_t & T_4 & \dot{m}_1 \end{bmatrix} \quad , \qquad (12.2)$$

and internal parameters were chosen:

➤   stagnation pressure recovery coefficient of the inner duct - σ_{bcs} ;
➤   isentropic efficiency of compressor - η_k ;
➤   isentropic efficiency of turbine - η_t ;
➤   stagnation pressure recovery coefficient of the combustion chamber σ_{et};
➤   mechanical efficiency η_m ,

$$\underline{x}^T = \begin{bmatrix} \sigma_{bcs} & \eta_k & \eta_t & \sigma_{et} & \eta_m \end{bmatrix} \quad . \qquad (12.3)$$

The diagnostic matrix determined by the above-mentioned method is:

$$\underline{\underline{D}} = \begin{bmatrix} -1,5675 & -0,0024 & 0,3510 & -0,0366 & -1,0352 \\ -1,2234 & 0,7539 & 0,1551 & -0,0224 & -0,1615 \\ 0,7739 & -0,4662 & -2,8423 & 5,0904 & 3,5961 \\ 2,4468 & -0,4765 & -0,3101 & 0,0447 & 0,3229 \\ -0,4395 & -0,3310 & 4,3752 & -6,9589 & -5,0412 \end{bmatrix} \quad . \qquad (12.4)$$

12.3 The Correlation-family test

12.3.1 Introduction

The correlation-family test of measurable external parameters of the examined system is one of the diagnostic methods based upon a statistical method.

Its goal is to establish the correlation of external parameters between themselves in case of anomalies or changes of internal ones [17]. The correlation coefficient characterizes the strength of stochastic interdependencies of the random variables [3]. Using the result of the analysis, which is the so called correlation-graph, sets of external parameters can be determined in which the parameters have got strong

correlation between themselves. One of the members of a family can be chosen for the measuring that the momentary technical state of the system can be determined in the easier way with adequate safety.

If one of the two parameters that have got strong positive correlation, changes in some direction, another one will change in the same direction most probably according to the correlation coefficient. (If correlation is strong by negative the changes have opposite directions.) In this case it is sufficient to measure one of them.

The correlation-family test investigates the stochastic interdependencies between the external parameters of the examined system. Using this method you can choose those parameters which should be measured for the optimal determination of technical state of the system.

Relation between phenomena is termed stochastic, if the course of one of them influences another, but not unambiguously [23]. The strength of stochastic relation of two random variables can be characterized by their correlation coefficient. The correlation coefficient of random variables with finite positive variance can be written in the

$$R(\eta, \mu) = \frac{M[(\eta - M(\eta))(\mu - M(\mu))]}{D(\eta)D(\mu)} \tag{12.5}$$

form [7], where:

M - mean;
D - variance.

If η and μ have no interdependence, then

$$R(\eta, \mu) = 0 \quad . \tag{12.6}$$

If

$$R(\eta, \mu) > 0 \quad , \tag{12.7}$$

η and μ have positive correlation, and for example we can deduce generally the inequation

$$\mu > M(\mu) \tag{12.8}$$

from the inequation

$$\eta > M(\eta) \quad , \tag{12.9}$$

and inversely, This means that the values of either parameter are bigger than their means. In case of negative correlation that is

$$R(\eta, \mu) < 0 \quad , \tag{12.10}$$

if inequation (12.9) exists, we can suppose that inequation

$$\mu < M(\mu) \tag{12.11}$$

exists too. Then deviations of either random variable from their means have got opposite signs.

It is important to note that the value of the correlation coefficient is always between -1 and +1 that is

$$-1 \leq R(\eta, \mu) \leq 1 \quad . \tag{12.12}$$

The correlation coefficient $R(\eta, \mu)$ can be estimated statistically by equation

$$r_{\eta\mu} = \frac{\sum_{i=1}^{n}\left(x_i - \sum_{j=1}^{n} x_j\right)\left(y_i - \sum_{j=1}^{n} y_j\right)}{\sqrt{\sum_{i=1}^{n}\left(x_i - \sum_{j=1}^{n} x_j\right)^2 \sum_{i=1}^{n}\left(y_i - \sum_{j=1}^{n} y_j\right)^2}} \qquad (12.13)$$

using the samples $x_1;\ x_2;\ ...\ x_n$ and $y_1;\ y_2;\ ...\ ;y_n$ which belong to variables η and μ [7].

This is termed the experimental correlation of the samples $x_1\ ;\ x_2\ ;\ ...\ x_n$ and $y_1\ ;\ y_2\ ;\ ...\ ;\ y_n$ too.

In case of several random variables the correlation coefficients r_{ij} determined by equation (12.13) can be arranged into the correlation-matrix

$$\underline{\underline{R}} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & & r_{2n} \\ \vdots & & & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nn} \end{bmatrix} \quad . \qquad (12.14)$$

The correlation-matrix is always symmetrical and the elements of its main diagonal is always equal to 1 that is:

$$r_{ii} = 1$$

and   (12.15)

$$r_{ij} = r_{ji} \quad .$$

12.3.2 Methodology

For the usage of correlation-family test we have to have enough data for statistical analysis. These data can be got by using the examined system or its model. In practical aspect, it is most suitable to use the linear or non-linear mathematical model of the system.

The first step is the determination of distribution of real values of non-measurable internal parameters. For this, we can use the data got during filling up the model or analysis of features of the system operation.

On the basis of distributions and statistical characteristics of the internal parameters the model can be generated. Using these results and equation (12.13) and (12.14), the correlation coefficients should be determined and arranged into the correlation-matrix.

The coefficient which has the most absolute value should be chosen and its parameters should be represented as the angular points of a graph. The correlation coefficient should be written at this edge.

The coefficient with the most absolute value of the rows and columns of the last two parameters should be chosen so that the ones which had been represented into the graph. have to be left out. This parameter and coefficient should be represented into the graph.

	A	B	C	D	E
A		-0,40	0,95	-0,80	-0,20
B	-0,40		0,60	0,15	0,70
C	0,95	0,60		-0,30	0,45
D	-0,80	0,15	-0,30		0,51
E	-0,20	0,70	0,45	0,51	

a.

	A	B	C	D	E
A		-0,40	0,95	-0,80	-0,20
B	-0,40		0,60	0,15	0,70
C	0,95	0,60		-0,30	0,45
D	-0,80	0,15	-0,30		0,51
E	-0,20	0,70	0,45	0,51	

b

	A	B	C	D	E
A		-0,40	0,95	-0,80	-0,20
B	-0,40		0,60	0,15	0,70
C	0,95	0,60		-0,30	0,45
D	-0,80	0,15	-0,30		0,51
E	-0,20	0,70	0,45	0,51	

c

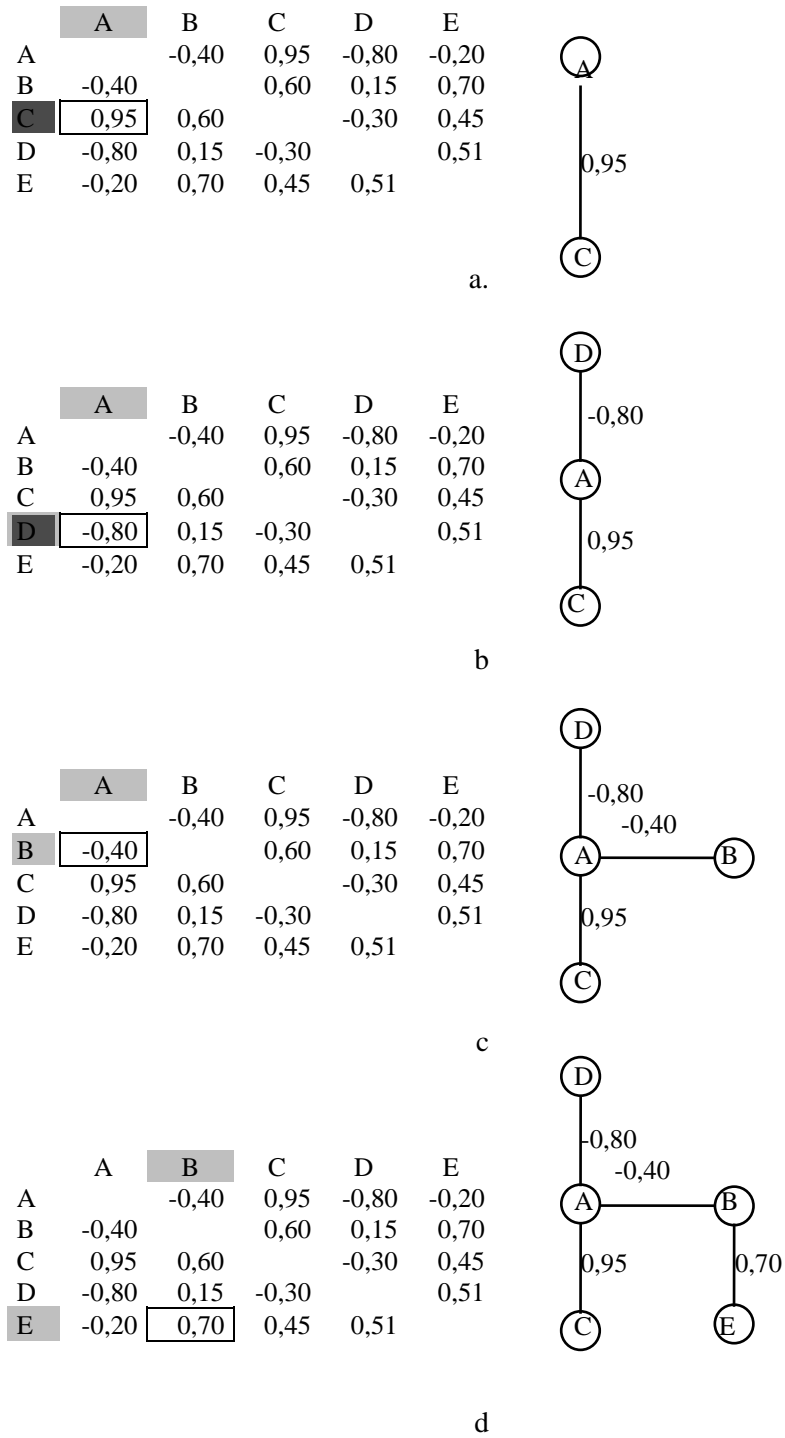|	A	B	C	D	E
A		-0,40	0,95	-0,80	-0,20
B	-0,40		0,60	0,15	0,70
C	0,95	0,60		-0,30	0,45
D	-0,80	0,15	-0,30		0,51
E	-0,20	0,70	0,45	0,51	

d

Figure 12.17. Creation of Correlation Graph

The correlation families should be separated so that the correlation coefficients of the graph in each families have to be more than the determined limit. One of the members of each families, chosen by the above-mentioned way, should be chosen that can be measured the most suitably. For the choice, you have to consider that the chosen parameter has to have strong correlation with another members of the family, it needs to be measured easily. Then it is a very important question that how many information given parameter has about momentary technical sate of the investigated system. Analyzing the result of sensitivity test, it can be determined (for example Chapter 12.3.4). For the choice of the most suitable measured parameter, other (not mathematical modeling) questions have to be taken into consideration depending on the system's tasks, works and construction.

One of the main questions of the correlation-family test is the determination of the limit for separation of families. If you use higher limit, you will get more families. In this case you can get more information about momentary technical state of the system, but more families mean more measured parameters. They require more technical and financial investment. If the used limit is lower, the number of families will decrease. In this case, technical investment will decrease, but the information which can be got about the system will or can decrease. Therefore determination of this limit requires careful consideration. You have to consider the feature of the examined system, the technical and financial possibilities and the needed accuracy of the technical state determination. It is important point of view that the consequence of troubles or the faulty work of the system (catastrophe or disturbance). It is generally suitable that the limit should be between 0,5-0,8 [12].

The other question of this test is the suitable number of statistical sample (excitation of model) for exact determination of correlation coefficients. For solution this problem the number of excitation should be grown increasingly until differences between the same elements of the matrices are decreased below a determined limit [14].

12.3.3 Case of the Pneumatic System of the Helicopter Mi-8

The correlation-family test of the pneumatic system of the helicopter Mi-8 has been carried out, using its linear diagnostic model set-up in the Chapter 11.1 [18]. For excitation of the model the following parameters were changed randomly:

Parameter	Distribution
force of the reducing spring of PU-7;	uniform
force of the small spring of PU-7;	normal
force of spring of UPO-3/2;	normal
ambient pressure;	uniform
tank pressure prior to braking;	uniform
clearances of brake-shoes;	uniform
stiffness of restoring springs of brake-shoes.	normal

Table 12.3. The Exciting Parameters

The number of excitations was grown increasingly till differences between the same elements of matrices decreases below 0,01. The correlation-matrix of the brake-system got by the above-mentioned analysis is:

$$\underline{\underline{R}} = \begin{array}{ccccccc} p_c & p_b & F_1 & F_2 & F_3 & F_4 & p_a \\ \begin{bmatrix} 1 & & & & & & \\ 0,921 & 1 & & & & & \\ 0,854 & 0,93 & 1 & & & & \\ 0,785 & 0,865 & 0,802 & 1 & & & \\ 0,851 & 0,927 & 0,86 & 0,812 & 1 & & \\ 0,914 & 0,958 & 0,926 & 0,865 & 0,928 & 1 & \\ -0,154 & -0,152 & -0,147 & -0,119 & -0,156 & -0,153 & 1 \end{bmatrix} \end{array}$$

(12.16)

The correlation-graph determined on the basis of the above matrix is shown in figure 12.18, where the measured parameters are signed by thick curves.
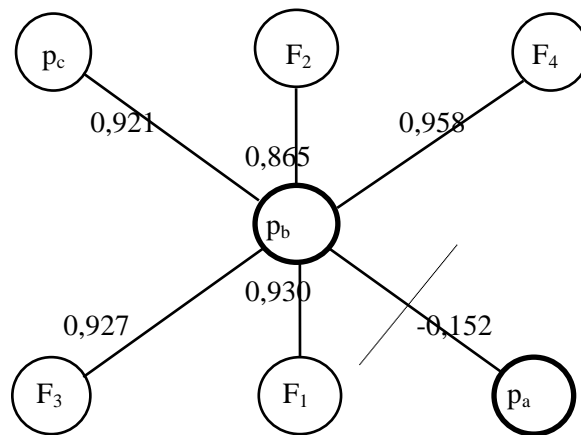

Figure 12.18. The Correlation Graph of the Pneumatic System of Helicopter Mi-8

The conclusions of this investigation are the following ones:

✈ the output parameters of the system have got strong correlation between themselves, except the tank pressure after braking

✈ measuring on only two parameters is sufficient, these ones are

- tank pressure after braking p_a ;

Its measuring is suitable for checking of the compressed air support sub-system too.

- pressure of brake-air p_b ;

Its measuring is suggested because its correlation with the other members of its family is strong and because it is a well-measurable parameter.

✈   gages of the helicopter Mi-8 measure the above-mentioned parameters, therefore their choice during the planning has been proven proper.

12.3.4 Case of the Starter Engine AI-9V

For the correlation-family analysis of starter-engine AI - 9V I used nonlinear thermodynamic mathematical model of the engine [19]. For excitation of the model the following parameters were changed randomly:

Parameter	Distribution
number of revolution	normal
efficiency of the compressor	exponential
efficiency of the turbine	exponential
efficiency of the combustion chamber	exponential
coefficient of pressure loss of the combustion chamber	exponential
coefficient of pressure loss of the inlet scoop	exponential
area of the inlet scoop	normal
mechanical efficiency	normal
flight level	uniform

Table 12.4. The Exciting Parameters

As you see from figure 12.19, the eleven chosen external parameters can be separated into three correlation-families, but two of them are one-member ones.

On the basis of correlation-family test, the gas temperature after the turbine T_4 should be measured. It characterizes the thermal load of the engine (mainly of the turbine), it has strong correlation with another members of its family, and its measuring can be carried out easily.

Only the member of another family that is the air pressure after the compressor p_2 is measured during main duty of the engine.

Only on the basis of correlation-family test, measuring of mass flow ratio to main engines $\dot{m}_1$ seems suitable. However from the technical point of view, it can be measured only by losing so resultant efficiency of the engine should decrease. According to another diagnostic investigation it can be established that this parameter has minimal information. In cases of each simulated failure of the engine, it decreases. Therefore it can be indicated only the fact of failure and aging of the engine and it cannot get information about their feature. On the basis of these reasons, you should disregard measuring of this parameter.

The correlation-matrix got by analysis mentioned above is:

$$
\underline{\underline{R}} =
\begin{array}{ccccccccccc}
\pi_K & T_2 & p_2 & \dot{m}_1 & T_3 & p_3 & q_t & \dot{m}_T & \pi_T & T_4 & p_4 \\
\end{array}
$$

$$
\underline{\underline{R}} =
\left[
\begin{array}{ccccccccccc}
1 & & & & & & & & & & \\
0{,}942 & 1 & & & & & & & & & \\
0{,}657 & 0{,}61 & 1 & & & & & & & & \\
-0{,}335 & -0{,}167 & 0{,}086 & 1 & & & & & & & \\
0{,}758 & 0{,}496 & 0{,}538 & -0{,}542 & 1 & & & & & & \\
0{,}173 & 0 & 0{,}647 & -0{,}05 & 0{,}438 & 1 & & & & & \\
0{,}739 & 0{,}5 & 0{,}581 & -0{,}438 & 0{,}93 & 0{,}561 & 1 & & & & \\
0{,}542 & 0{,}235 & 0{,}673 & -0{,}263 & 0{,}756 & 0{,}839 & 0{,}907 & 1 & & & \\
0{,}167 & 0 & 0{,}649 & -0{,}048 & 0{,}422 & 0{,}999 & 0{,}547 & 0{,}83 & 1 & & \\
0{,}722 & 0{,}452 & 0{,}505 & -0{,}468 & 0{,}993 & 0{,}425 & 0{,}919 & 0{,}741 & 0{,}408 & 1 & \\
0{,}194 & 0 & 0{,}634 & -0{,}034 & 0{,}494 & 0{,}994 & 0{,}606 & 0{,}86 & 0{,}991 & 0{,}492 & 1 \\
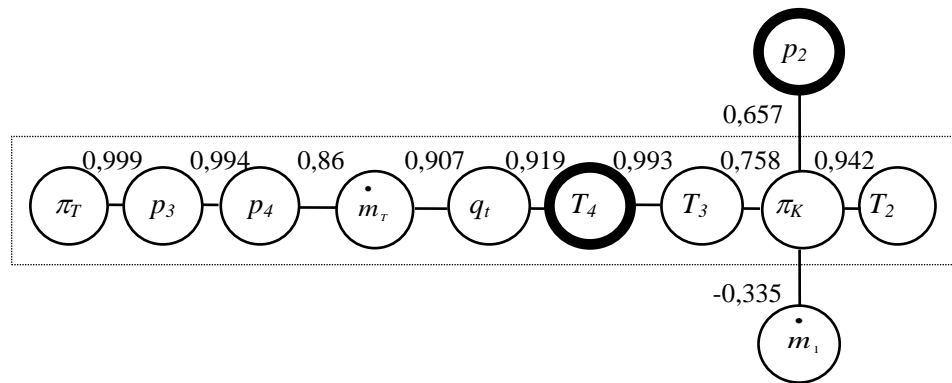\end{array}
\right]
$$

$$(12.17)$$

Figure 12.19. The Correlation Graph of the Engine AI-9V

The correlation-graph determined on the basis of the matrix above is shown in figure 12.19, where the measured parameters are signed by thick curves.

The results of the above investigation have justified the decisions of designing engineers.

12.4. Investigation of Prohibited Duty

Using mathematical model, the prohibited duties of the examined system can be investigated without its break-down or damage. This chapter will show an example of this very interesting possibility of use of mathematical models.

The starter-engine AI - 9V has two duties that must not be used at same time. They are:

„air-topping" ;
The compressed air is tapped from after-compressor space to starter-turbines of main engine of the helicopter to start them.

„generator" .
In this case the turbine propels the compressor and the starter-generator of AI - 9V to supply the direct current electrical system of the helicopter.

When the author set up the thermodynamical model of the starter-engine AI-9V, I remembered this practical question in that he was interested when he operated helicopters MI - 24 D. Therefore, modifying the original mathematical model, the author simulated this duty. For modeling that turbine propels the compressor and the starter-generator, the following equation was used:

$$P_t \eta_m - P_k - P_a - P_{gen} = 0 \qquad (12.18)$$

instead of equation (11.19).

Parameter	Results of		Permissible (nominal) value
	modified	original	
	model		
temperature after the compressor	443 K	434 K	
pressure after the compressor	307889 Pa	294344 Pa	
temperature before the turbine	1251 K	1164 K	
pressure before the turbine	289425 Pa	276684 Pa	
temperature after the turbine	1050 K	979 K	1023 K (750 °C)
pressure after the turbine	114020 Pa	112877 Pa	
mass flow rate to main engines	0,403 kg s^{-1}	0,399 kg s^{-1}	0,4 kg s^{-1}

Table 12.5. Results of Prohibit Duty Modeling

The results of the modified model (see table 12.5) show the following:

✈   the temperature before the turbine T_3 has increased by 36 ° C;

✈   the temperature after the turbine T_4 has increased by 31 ° C and it has been more than its maximal permissible value.

On the basis of the above mentioned facts, it can be established that the thermal load of the turbine, which is the most structural and material problem of gas-turbine engines, increased considerably. This increased temperature and thermal load can damage the rotor blades of the turbine. Therefore, it can be stated that the usage of this investigated operating regulation is justified.

It is important to mention that the above mentioned investigation could be done without the damage of turbine rotor blades, and only by using the mathematical model of the starter-engine. This possibility of use is one of the most important advantages of the mathematical modeling.

APPLICATION OF THE MODEL FOR IDENTIFICATION OF THE TECHNICAL STATE OF THE SYSTEM

13.1. Introduction

The linear mathematical diagnostic model can be used to estimate the momentary technical state of the examined system as well. For this investigation, the parameters need to be classified that the vector $\delta \underline{y}$ includes only the detectable - external - parameters, and the vector $\delta \underline{x}$ includes the non-measurable - internal - ones [13].

On the basis of the above-determined vectors and their coefficient matrices $\underline{\underline{A}}$ and $\underline{\underline{B}}$ should be determined

$$\underline{\underline{A}}\delta \underline{y} = \underline{\underline{B}}\delta \underline{x} \quad . \tag{13.1}$$

Knowing the elements of the coefficient matrices and values of internal parameters (elements of vector $\delta \underline{y}$) of the equation (13.1), the values of the internal parameters that is the elements of the vector $\delta \underline{x}$ can be determined.

13.2. Methods of Identification

The vector $\delta \underline{x}$ can be estimated by two methods.

13.2.1. Case of Quadratic Coefficient Matrix

If the coefficient matrix of the internal parameters is a quadratic one that can be inverted, you can use the diagnostic matrix

$$\underline{\underline{D}} = \underline{\underline{A}}^{-1}\underline{\underline{B}} \tag{13.2}$$

and the equation (13.1) can be rearranged to

$$\delta \underline{y} = \underline{\underline{A}}^{-1}\underline{\underline{B}}\delta \underline{x} = \underline{\underline{D}}\delta \underline{x} \quad . \tag{13.3}$$

In this case you have to find the vector $\delta \underline{x}$ that satisfies the equation

$$\underline{y} - \underline{\underline{D}}\delta \underline{x} = \underline{0} \quad . \tag{13.4}$$

Therefore the minimum of the equation should be estimated

$$f(\underline{x}) = \underline{y} - \underline{\underline{D}}\delta \underline{x} \tag{13.5}$$

using any numerical searching algorithm.

13.2.2. Case of Non-Quadratic Coefficient Matrix

If the coefficient matrix of the internal parameters $\underline{\underline{A}}$ is not quadratic so it cannot be inverted, you have to introduce the auxiliary vector

$$\underline{u} = \underline{\underline{A}}\delta\underline{y} \quad . \tag{13.6}$$

Therefore the equation (13.1) can be rearranged to

$$\underline{u} = \underline{\underline{B}}\delta\underline{x} \quad . \tag{13.7}$$

Then, the vector $\delta\underline{x}$ which satisfied the equation should be estimated

$$\underline{u} - \underline{\underline{B}}\delta\underline{x} = \underline{0} \tag{13.8}$$

that is the minimum of the scalar - vector function

$$f(\underline{x}) = \left|\underline{u} - \underline{\underline{B}}\delta x\right| \tag{13.9}$$

should be estimated.

The vector $\delta\underline{x}$ that satisfies the equation (13.5) or (13.9) can be estimated by using any search of optimum method.

13.3. Search of Optimum Methods

In the technical practice the following search of optimum methods are used basically:

✈   GRADIENT METHOD;
✈   RANDOM METHOD;
✈   GAUSS-SEIDEL METHOD

The gradient method has been introduced in the Chapter 10.4.2.

The disadvantage of this method is that it can run to a local minimum point. Therefore the Gradient method should be used by different initial points or with any other optimum search method.

13.3.1. The RANDOM Method

The essence of the RANDOM-method is that a domain around the „zero-th approximate point" is designed and point in this domain are chosen randomly. The function value of these points is determined. The point that has the most minimal function value should be chosen and a less domain around it should be designed. The above-mentioned procedure should be repeated until the function value decreases below the determined limit.
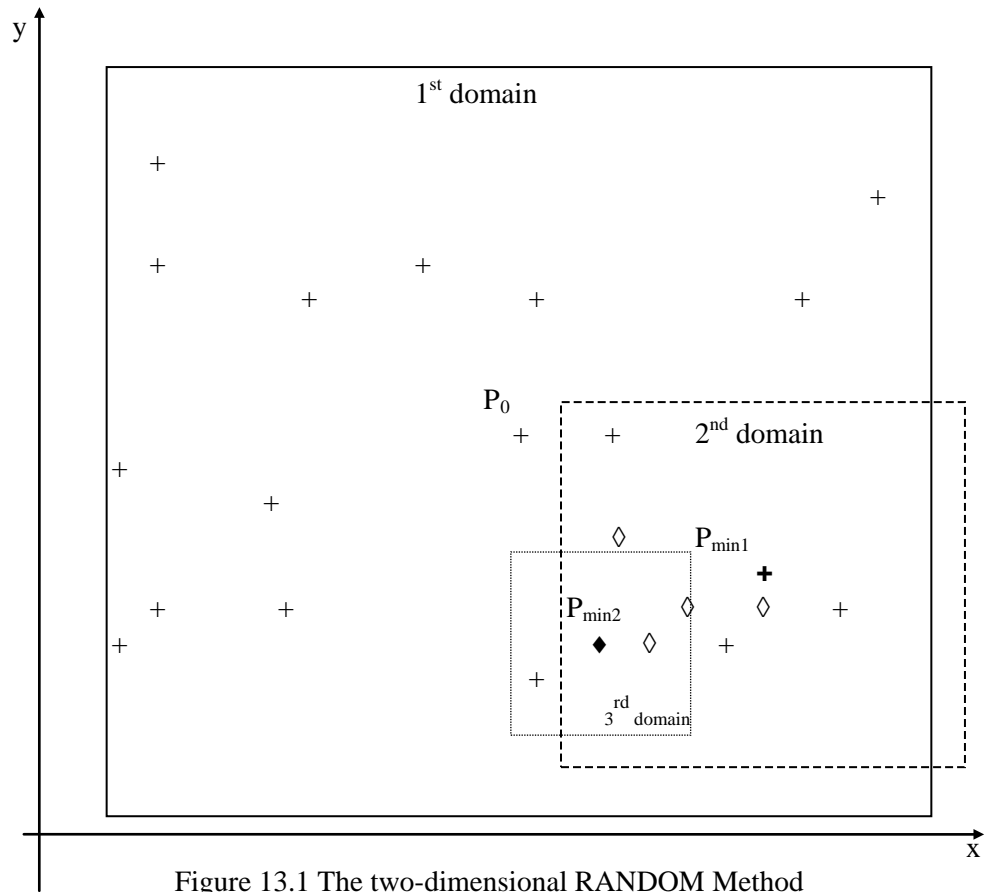
Figure 13.1 The two-dimensional RANDOM Method

13.3.2. GAUSS-SEIDEL Method

The GAUSS-SEIDEL method is one of the most known relaxation ones to solve the

$$\underline{\underline{C}}\underline{z} - \underline{r} = \underline{0} \tag{13.10}$$

type linear system of equation. Its principle is the minimum points along the axes of the state space are searched successively.

Using this method, the elements of vector $\underline{z}$ should be modified one by one in order to let the actual element of vector $\underline{d}_k$ in equation:

$$\underline{\underline{C}}\underline{z} - \underline{r} = \underline{d}_k \tag{13.11}$$

equal to zero.

In this case the following iteration equations should be used:

$$\underline{z}_{k+1} = \underline{z}_k + \underline{v}_k \tag{13.12}$$

$$\underline{v}_k^T = \begin{bmatrix} 0 & \dots & 0 & v_k & 0 & \dots & 0 \end{bmatrix} \tag{13.13}$$

$$v_k = -\frac{d_k}{c_{kk}} \tag{13.14}$$

13.4. The Case of the Pneumatic System of the Helicopter Mi-8

In case of the pneumatic system of the helicopter Mi-8 the vectors $\delta \underline{x}$ and $\delta \underline{y}$ were separated so that the vector of external parameters includes ones that are measurable even now. I applied this method, because it has to be used for a helicopter that has not been designed for the on-condition maintenance.

$$\delta \underline{y}^T = \begin{bmatrix} \delta p_H & \delta p_b & \delta p_a & \delta z_1 & \delta z_2 & \delta z_3 & \delta z_4 & \delta p_t \end{bmatrix} \quad , \quad (13.15$$

$$\delta \underline{x}^T = \begin{bmatrix} \delta p_c & \delta F_{s1} & \delta F_{s2} & \delta F_{s3} & \delta s_1 & \delta F_1 & \delta s_2 & \delta F_2 & \delta s_3 & \delta F_3 & \delta s_4 & \delta F_4 \end{bmatrix}$$
$$(13.16)$$

Knowing the external and internal parameters their coefficient matrices have been determined.

$$\underline{\underline{A}} = \begin{bmatrix} -K_4 & 0 & K_3 & 0 & 0 & 0 & 0 & 0 \\ K_7 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -K_9 & -K_8 & 0 & K_{10} & 0 & 0 & 0 & 0 \\ -K_9 & -K_8 & 0 & 0 & K_{10} & 0 & 0 & 0 \\ -K_9 & -K_8 & 0 & 0 & 0 & K_{10} & 0 & 0 \\ -K_9 & -K_8 & 0 & 0 & 0 & 0 & K_{10} & 0 \\ -K_{15} & K_{14} & 1 & -K_{16} & -K_{16} & -K_{16} & -K_{16} & -K_{12} \end{bmatrix}$$
$$(13.17)$$

$$\underline{\underline{B}} = \begin{bmatrix} -1 & K_1 & -K_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ K_5 & 0 & 0 & -K_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -K_{11} & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -K_{11} & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -K_{11} & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -K_{11} & -1 \\ -K_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$(13.18)$$

Since coefficient matrix of the internal parameters was not quadratic one, the equation (13.9) and the gradient method were used. The result of the investigation can be seen in the following tables and figures.

Working time of the helicopter chosen for investigation was 52 hours 06 minutes (649.10 - 701.16) from 19th June to 20th November. The following figures show the test results depending on operating hours and depending on calendar time.

Date:	09.19.			
Tail Number:	XXX			
Flying Hours:	673 . 02			
Ambient pressure:		[torr]	753,5	
Right Break-Shoes' Clearance:		[mm]	0,30,4	
Left Break-Shoes' Clearance:		[mm]	0,40,45	
Pressures	[atm]			
Prior to Breaking	47,5	45	43,5	42
After Breaking	45	43,5	42	40
Break	33,5	32	32,5	34

Table 13.1. Measured Data

Estimated Differences [‰]		
1	p_c	-60,2181600
2	F_{s1}	1,6574380
3	F_{s2}	-15,6758500
4	F_{s3}	-15,6471700
5	s_1	-16,0464700
6	F_1	-11,8686600
7	s_2	0,0868354
8	F_2	-14,9381200
9	s_3	-0,1951021
10	F_3	-14,4642500
11	s_4	-0,2006741
12	F_4	-14,7801600
Break effort		0,4324500
Break Asymmetry		0,1037500

Table 13.2 Result of State/Estimation

δF_Σ [‰]
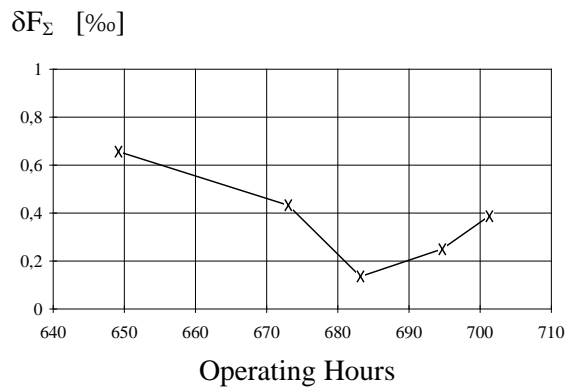
Operating Hours

Figure 13.2. Decrease of the Brake-Effort Depending on Operating Hours

δF_{Σ} [‰]
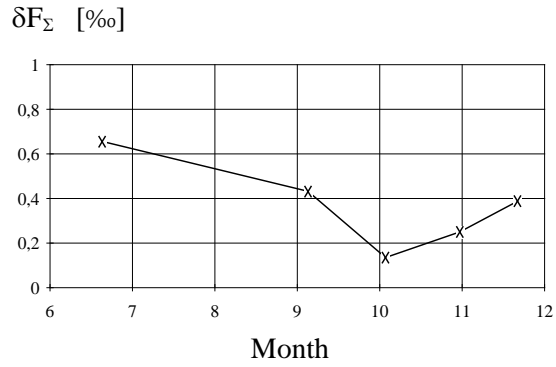
Figure 13.3. Decrease of the Brake-Effort Depending on Calendar Time
(of the Investigating Year)

δF_{Δ} [‰]
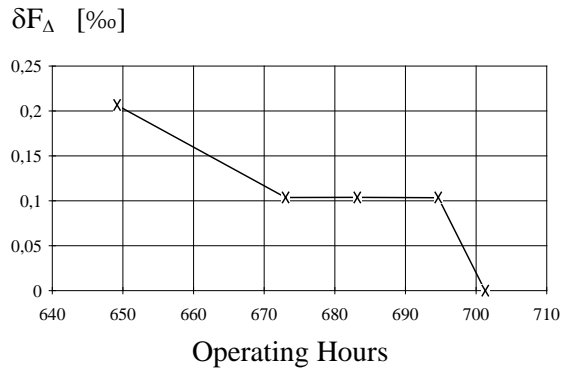
Operating Hours

Figure 13.4. Brake-Asymmetry Depending on Operating Hours
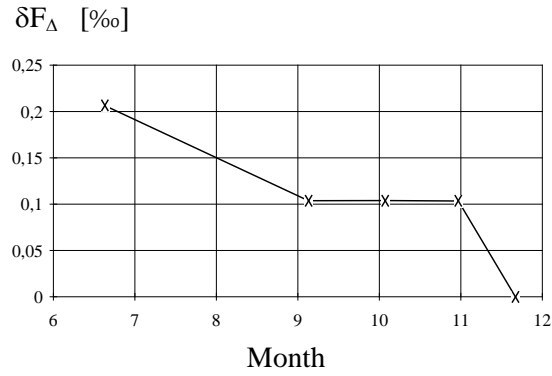
δF_{Δ} [‰]

Month

Figure 13.5. Brake-Asymmetry Depending on Calendar Time
(of the Investigating Year)

APPLICATION OF THE MODEL FOR MANAGEMENT OF THE OPERATIONAL PROCESS

14.1. Introduction

Let general parameter η characterize the technical state of the investigated system (see Figure 14.1). If the value of parameter η meets the η_{br} brake value, the system will break-down. Let τ be the parameter which characterizes the performance of the system. For example, this parameter can be the effective calendar time, effective flying hours (in case of the airframe), number of landings (in case of landing gear systems), or number of starts (in case of gas-turbine engines) from installation or the last overhaul.
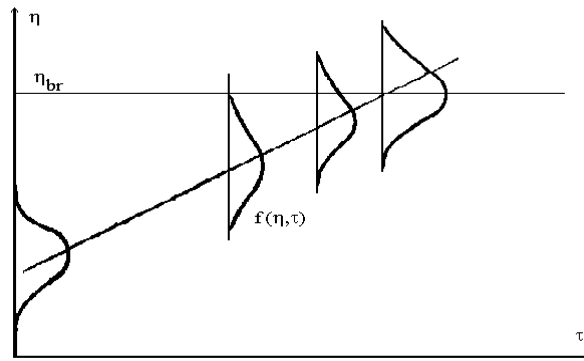
Figure 14.1 The Wearing-out Process

In this case the wearing-out process of the system, that is the $\eta(\tau)$ stochastic function can be characterized by:

$\widehat{\eta}(\tau)$ -   the expected value function of the parameter η ;

$f(\eta, \tau)$ -   the density function of parameter η .

Then the probability of good working state of the system:

$$P_{gw}(\tau) = P(\eta_{br} > \eta(\tau)) = \int_{-\infty}^{\eta_{br}} f(\eta, \tau) d\tau \qquad . \qquad (14.1)$$

The process of changing of parameter η can be described by:

$\overset{\circ}{\eta}(\tau)$ -   the changing velocity of the parameter η ;

$\varphi\left(\overset{\circ}{\eta},\tau\right)$ -   the density function of the changing velocity.

Then the "failure changing velocity" of parameter η is:

$$\overset{\circ}{\eta}_{br}(\tau,\Delta\tau)=\frac{\eta_{br}-\eta(\tau)}{\Delta\tau} \quad \text{if} \quad \eta_{br}>\eta(\tau) \tag{14.2}$$

and the probability of good working state of the system in the interval (η , $\eta+\Delta\eta$):

$$P_{gw}(\tau,\Delta\tau)=P\left(\overset{\circ}{\eta}_{br}(\tau)>\overset{\circ}{\eta}(\tau)\right)=\int_{-\infty}^{\overset{\circ}{\eta}_{br}}\varphi\left(\overset{\circ}{\eta},\tau\right)d\overset{\circ}{\eta} \tag{14.3}$$

supposing that the system is ready to service at the start of the investigated performance interval.

14.2. Operational Management Method

For exact and manageable comparison of different technical states and management of the operational process, the so called "leader parameter" should be introduced. The leader parameter can be the most important one for the operation and the maintenance of the system. This should be one of parameters estimated above (see Chapter 13) or a parameter which can be determined directly from internal ones. For example, the leader parameter can be the thrust or useful power in case of gas-turbine engines.

Depending on the momentary values of the leader parameter and its velocity, the needed service work can be decided. For decision, permissible value and permissible velocity of the leader parameter should be determined on the basis of its breakdown value and permissible probability of risk.

Knowing the breakdown value η_{br} of the parameter η and performance interval between checks $\Delta\tau$, the permissible value η_p and the permissible changing velocity to ready for working should be determined. Supposing that:

→   the change of the parameter η on interval $\Delta\tau$ (see Figure 14.2) is a linear one;
→   the density function of the changing velocity is independent on working performance of the system.

14.2.1. Determination of Permissible Velocity

If the value of the parameter η reaches the permissible value η_p at   the i-th checking and it changes with

$$\overset{\circ}{\eta}>\frac{\Delta\eta}{\Delta\tau}$$

velocity, the parameter η is going to reach the breakdown value η_{br} before the next $(i+1$-th) check, in other words the operated system will break-down.
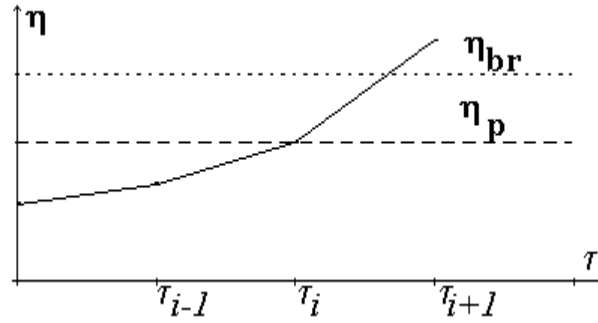

Figure 14.2. Determination of the Permissible Parameter Values

Therefore, permissible velocity of the parameter η to ready for working is:

$$\overset{\circ}{\eta}_p = \frac{\Delta\eta}{\Delta\tau} \qquad . \tag{14.4}$$

The probability of breakdown is:

$$P_{br}(\Delta\tau, \Delta\eta) = P\left(\overset{\circ}{\eta} > \overset{\circ}{\eta}_{br}\right) = 1 - P\left(\overset{\circ}{\eta} \leq \overset{\circ}{\eta}_{br}\right) = 1 - \int_{-\infty}^{\overset{\circ}{\eta}_{br}} \varphi(\overset{\circ}{\eta})d\overset{\circ}{\eta} \tag{14.5}$$

Knowing the permissible probability of risk Q (permissible probability of breakdown), it is substituted into equation (14.5), equation

$$Q = P_{br}(\Delta\tau, \Delta\eta) = 1 - \int_{-\infty}^{\overset{\circ}{\eta}_p} \varphi(\overset{\circ}{\eta})d\overset{\circ}{\eta} \tag{14.6}$$

is got.

14.2.2. Determination of Permissible Value

If the density function of velocity $\overset{\circ}{\eta}$ cannot be determined by statistical method, the usage of one of the well-known density functions is suitable. For example:

- UNIFORM distribution:

$$\varphi(\overset{\circ}{\eta}) = \frac{1}{\overset{\circ}{\eta}_{max} - \overset{\circ}{\eta}_{min}} = \frac{1}{\Delta\eta} \qquad (\text{if } \overset{\circ}{\eta}_{max} > \overset{\circ}{\eta} > \overset{\circ}{\eta}_{min}) \tag{14.7}$$

Then

$$Q = 1 - \int_{-\infty}^{\overset{\circ}{\eta}_p} \frac{1}{\Delta\overset{\circ}{\eta}} d\overset{\circ}{\eta} = 1 - \frac{\overset{\circ}{\eta}_p}{\Delta\overset{\circ}{\eta}} = 1 - \frac{\Delta\eta}{\Delta\tau\Delta\overset{\circ}{\eta}} \qquad , \tag{14.8}$$

that is

$$\Delta\eta = (1 - Q)\Delta\tau\Delta\overset{\circ}{\eta} \qquad . \qquad (14.9)$$

-   EXPONENTIAL distribution:

$$\varphi(\overset{\circ}{\eta}) = \lambda e^{-\lambda\eta} \qquad (\text{if } \overset{\circ}{\eta} > 0) \qquad . \qquad (14.10)$$

Then

$$Q = 1 - e^{-\lambda\overset{\circ}{\eta}_p} \qquad , \qquad (14.11)$$

and

$$\Delta\eta = -\frac{\ln(1 - Q)}{\lambda}\Delta\tau \qquad . \qquad (14.12)$$

-   NORMAL (GAUSS) distribution:

$$\varphi(\overset{\circ}{\eta}) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(\overset{\circ}{\eta}-m)^2}{2\sigma^2}} \qquad . \qquad (14.13)$$

In this case simpler solution cannot be got like to above ones which is deduced easily by any algebraic way. Therefore, on the basis of its variance and expected value, transforming the normal distribution to the standard normal one, the permissible velocity of parameter η and parameter interval $\Delta\eta$ can be determined.

The permissible value of the parameter η to ready for working:

$$\eta_p = \eta_{br} - \Delta\eta \qquad . \qquad (14.14)$$

If momentary values η and $\overset{\circ}{\eta}$ smaller than those permissible ones determined by equation (14.14) and (@6.4), the system will not break down till the next check with probability of least *1-Q*.

14.3. Case of Pneumatic System of Helicopter Mi-8

Using result of the state-estimation of pneumatic break-system of helicopter Mi-8 depicted in Chapter 13, the operation management method and process will be shown.

The decrease of the brake-effort and brake asymmetry were chosen as leader parameters. To determine the permissible value and the velocity of this leader parameters,

$$Q = 0,025$$

the permissible probability of risk was used.

The quantity of data is not sufficient for statistical estimation of their distribution. Therefore, for determination of the permissible value and velocity of the resultant brake-effort, the density of its changing velocity is supposed as an uniform one - see equations (14.7); (14.8) and (14.9).

Flying hours of the helicopter chosen for investigation was 52 hours 06 minutes (649.10 - 701.16) from 19th June to 20th November. The following figures show the test results depending on the flying hours and depending on the calendar time.

Table 14.1. shows the estimated values and permissible values of leader parameters and their changing velocities. The following figure shows the changing velocities of leader parameters depending on flying hours and calendar times. Their momentary values are shown in Chapter 13. (see Figures 13.2. - 13.5.).

Number of Check	I.	II.	III.	IV.	V.
Data	06. 19.	09. 19.	10. 02.	10. 30.	11. 20.
Flying Hours	649. 10	673. 02	683. 10	694. 39	701. 06
Δt_c [day]	—––—	77	28	28	21
Δt_w [f.hours]	—––—	23,87	10,13	09,48	06,62
Break effort[‰]	0,65655	0,43245	0,13550	0,25030	0,38770
Permissible Value [‰]	91,5				
Dep. on Cal.Time [day^{-1}]	—––—	-2,91 10^{-6}	-1,06 10^{-5}	4,10 10^{-6}	6,54 10^{-6}
Perm. Value [day^{-1}]	8,10 10^{-5}				
Dep. on F. Hours. [f.h.$^{-1}$]	—––—	-9,42 10^{-6}	-2,93 10^{-5}	1,21 10^{-5}	2,08 10^{-5}
Perm. Value [f.h.$^{-1}$]	4,35 10^{-5}				
Break Asym. [‰]	0,20645	0,10375	0,10390	0,10345	0,00000
Perm. Value[‰]	25				
Dep. on Cal.Time [day^{-1}]	—––—	-1,33 10^{-6}	5,36 10^{-9}	-1,61 10^{-8}	-4,93 10^{-6}
Perm. Value [day^{-1}]	2,95 10^{-5}				
Dep. on F. Hours. [f.h.$^{-1}$]	—––—	-4,30 10^{-6}	1,48 10^{-8}	-4,75 10^{-8}	-1,56 10^{-5}
Perm. Value [f.h.$^{-1}$]	1,90 10^{-5}				
Notice		good	good	good	good

Table 14.1. Result of the Operation Management Method

$$\frac{d\delta F_{\Sigma}}{dt_w} \text{ [f hour }^{-5}]$$
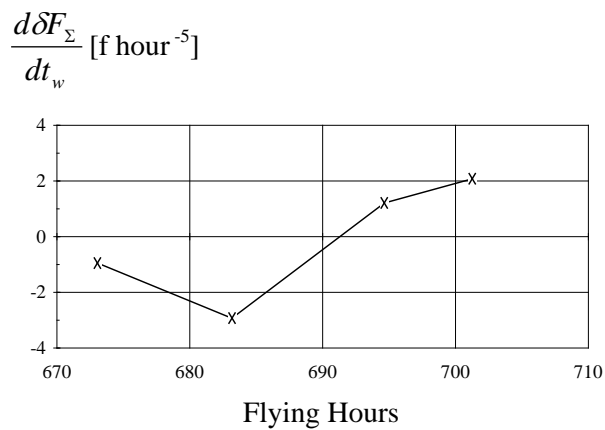
Figure 14.3. Changing Velocity of the Brake-Effort Depending on Flying Hours

$$\frac{d\delta F_{\Sigma}}{dt_c} \ [\text{day}^{-6}]$$
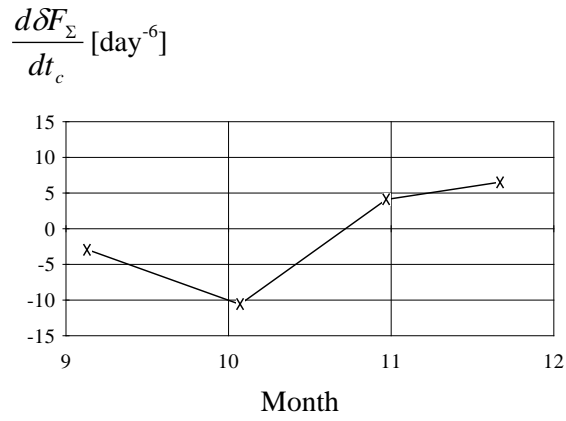
Figure 14.4. Changing Velocity of the Brake-Effort Depending on Calendar Time (of the Investigating Year)

$$\frac{d\delta F_{\Delta}}{dt_w} \ [\text{f.hour}^{-5}]$$
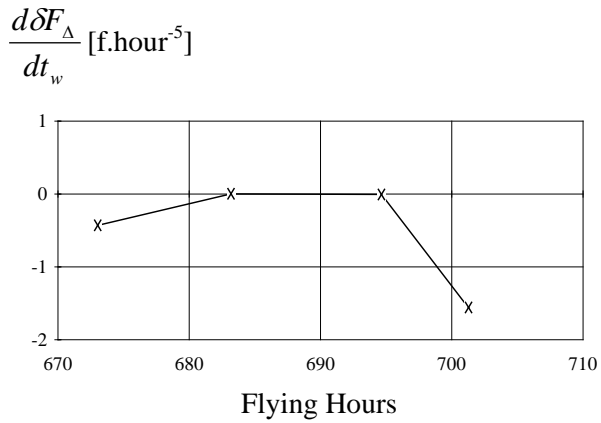
Figure 14.5. Changing Velocity of Brake-Asymmetry Depending on Flying Hours

$$\frac{d\delta F_{\Delta}}{dt_c} \ [\text{day}^{-6}]$$

Figure 14.6. Changing Velocity of Brake-Asymmetry Depending on Calendar Time (of Investigating Year)

CHAPTER 15

APPLICATION OF MODELS
TO INVESTIGATE EFFECTS
OF MANUFACTURING ANOMALIES

15.1. Introduction

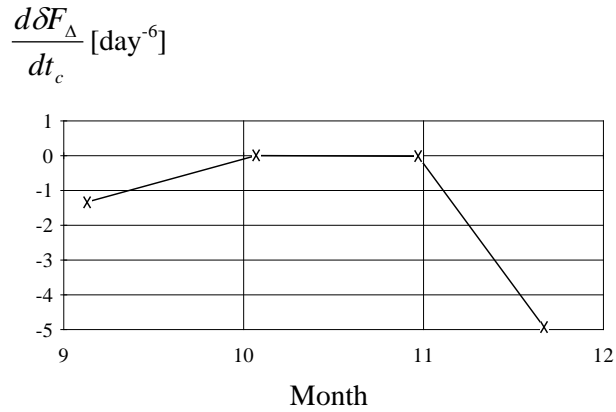During the design of new aircraft system the determination of manufacturing tolerances is a very important task. When the manufacturing tolerances of system components are determined, the influences of their probable manufacturing anomalies on the external parameters of the system should be investigated. This chapter will show the methodology the usage of mathematical diagnostic models to investigate influences of manufacturing anomalies and its inverse method.

It is possible that system parameter values are inadequate, but every unit and element of the system meet its own requirements. Because of the manufacturing tolerances of units or elements have been determined incorrectly.

It is possible too, that working requirement of the system determine strictly the system parameter tolerances. For example, such system requirements can be velocity or acceleration of the piston of hydraulic servo actuator, in the flight mechanical point of view. These external parameters should require the tolerances of internal parameters of the system strictly.

Using linearized mathematical diagnostic model of the given system the problems mentioned above can be investigated and solved.

Manufacturing anomalies of internal parameters can be characterized by their densities, expected values and variances. These random characteristics determine the densities, expected values and variances of external parameters so their manufacturing anomalies.

For investigation, the following matrices and vectors should be introduced [11]:

✈   Matrix of nominal values of the internal parameters;

$$\underline{\underline{X}} = \begin{bmatrix} x_{1_{nom}} & 0 & \ldots & 0 \\ 0 & x_{2_{nom}} & & \vdots \\ \vdots & & & \vdots \\ 0 & \ldots & \ldots & x_{p_{nom}} \end{bmatrix} \tag{15.1}$$

✈   Matrix of nominal values of the external parameters;

$$\underline{\underline{Y}} = \begin{bmatrix} y_{1_{nom}} & 0 & \dots & 0 \\ 0 & y_{2_{nom}} & & \vdots \\ \vdots & & & \vdots \\ 0 & \dots & \dots & y_{k_{nom}} \end{bmatrix} \qquad (15.2)$$

✈   Vector of measured variances of internal parameters;

$$\hat{\underline{x}}^T = \begin{bmatrix} \hat{x}_1 & \hat{x}_2 & \dots & \hat{x}_p \end{bmatrix} \qquad (15.3)$$

✈   Vector of measured expected values of internal parameters.

$$\tilde{\underline{x}}^T = \begin{bmatrix} \tilde{x}_1 & \tilde{x}_2 & \dots & \tilde{x}_p \end{bmatrix} \qquad (15.4)$$

15.2. Case of the "Gauss Distributions"

Now suppose that manufacturing anomalies of the internal parameters are interdependent random variables with normal distribution.

In this case the expected values of internal parameters are the means of their tolerance zones. It is important to mention, if the tolerance zones are asymmetric, the expected value will not be equal to the nominal value of the given parameters.

The variance of this parameters should be determined as a sixth parts of tolerance zones due to the so called "3σ-rule". Because the random variables of normal distribution with expected value m and variance σ will fall "practically certainly" in the (m-3σ, m+3σ) interval - its probability in fact is 0,9973 [23].

15.2.1. Determination of Variances

To determine the variances of external parameters, the vector of relative variances of interval parameters should be determined by equation

$$\delta\hat{\underline{x}} = \underline{\underline{X}}^{-1}\hat{\underline{x}} \qquad . \qquad (15.5)$$

Using the diagnostic matrix of the investigated system, the vector of relative variances of external parameters is:

$$\delta\hat{\underline{y}} = \underline{\underline{D}}\delta\hat{\underline{x}} = \underline{\underline{X}}^{-1}\underline{\underline{D}}\hat{\underline{x}} \qquad . \qquad (15.6)$$

Knowing the nominal values of the external parameters, the vector of their measured variances should be determined by following equation:

$$\hat{\underline{y}} = \underline{\underline{Y}}\delta\hat{\underline{y}} = \underline{\underline{X}}^{-1}\underline{\underline{D}}\underline{\underline{Y}}\hat{\underline{x}} \qquad . \qquad (15.7)$$

Introduce the "measured diagnostic coefficient matrix":

$$\underline{\underline{S}} = \underline{\underline{X}}^{-1}\underline{\underline{D}}\underline{\underline{Y}} \qquad , \qquad (15.8)$$

the equation (15.7) can be simplified:

$$\hat{\underline{y}} = \underline{\underline{S}}\delta\hat{\underline{x}} \qquad . \qquad (15.9)$$

15.2.2. Determination of Expected Values

To determine expected values of external parameters, the vector of relative expected values of integral parameters should be determined. Because the diagnostic model describes interdependencies between relative changes of internal and external parameters, this vector should show the relative values of difference between measured expected and nominal values to nominal ones. Therefore, using the *p*-dimensional summation vector $\underline{e}_p$ (every element of the vector is equal to 1):

$$\delta \widetilde{x} = \frac{\widetilde{x} - x_{nom}}{x_{nom}} \quad ,$$

that is   (15.10)

$$\delta \underline{\widetilde{x}} = \underline{\underline{X}}^{-1}(\underline{\widetilde{x}} - \underline{\underline{X}}\underline{e}_p) \quad .$$

Knowing the diagnostic matrix, the vector of relative expected values of external parameters should be determined by equation

$$\delta \underline{\widetilde{y}} = \underline{\underline{D}}\delta \underline{\widetilde{x}} = \underline{\underline{X}}^{-1}\underline{\underline{D}}(\underline{\widetilde{x}} - \underline{\underline{X}}\underline{e}_p) \quad . \tag{15.11}$$

Then, using the matrix of nominal values of external parameters and the *k*-dimensional summation vector $\underline{e}_k$, the vector of measured expected values of external parameters should be determined:

$$\underline{\widetilde{y}} = \underline{\underline{Y}}\delta \underline{\widetilde{y}} + \underline{\underline{Y}}\underline{e}_k = \underline{\underline{X}}^{-1}\underline{\underline{D}}\underline{\underline{Y}}(\underline{\widetilde{x}} - \underline{\underline{X}}\underline{e}_p) + \underline{\underline{Y}}\underline{e}_k \quad . \tag{15.12}$$

Applying the measured diagnostic coefficient matrix $\underline{\underline{S}}$ - see equation (15.8) - the equation (15.12) can be simplified:

$$\underline{\widetilde{y}} = \underline{\underline{S}}(\underline{\widetilde{x}} - \underline{\underline{X}}\underline{e}_p) + \underline{\underline{Y}}\underline{e}_k \quad . \tag{15.13}$$

Knowing the variances and the expected values of external parameters, their "manufacturing tolerance zones" that are result of manufacturing anomalies of units and elements of the system can be determined. These interval should be determined using the „3σ-rule" mentioned above, that is the vector of their minimum values:

$$\underline{y}_{min} = \underline{\widetilde{y}} - 3\underline{\hat{y}} \quad , \tag{15.14}$$

and vector of their maximum values:

$$\underline{y}_{max} = \underline{\widetilde{y}} + 3\underline{\hat{y}} \quad . \tag{15.15}$$

15.3. The Inverse Method

It is possible, that task and work of given system limit strictly output parameter values and their tolerances determined by equations (15.14) and (15.15) cannot meet these requirements. Then the manufacturing tolerances of internal parameters have to be determined on the basis of the required tolerances of the external system parameters. This task can be solved by the inverse method of the problem.

The required variances of the internal parameters should be determined by equation (15.9). The vector $\underline{\hat{x}}$ that satisfies the equation

$$f(\underline{\hat{x}}) = (\underline{\hat{y}} - \underline{\underline{S}}\,\underline{\hat{x}})^2 = 0 \tag{15.16}$$

should be estimated by using any search of optimum method.

For determination of required expected values of internal parameters the equations (15.13) should be rearranged

$$\underline{\tilde{y}} - \underline{\underline{Y}}\underline{e}_k = \underline{\underline{S}}(\underline{\tilde{x}} - \underline{\underline{X}}\underline{e}_p) \tag{15.17}$$

and auxiliary vectors

$$\underline{u} = \underline{\tilde{y}} - \underline{\underline{Y}}\underline{e}_k \tag{15.18}$$

and

$$\underline{v} = \underline{\tilde{x}} - \underline{\underline{X}}\underline{e}_p \tag{15.19}$$

should be introduced. Then the equation (15.17) will modify to the following form:

$$\underline{u} = \underline{\underline{S}}\underline{v} \qquad . \tag{15.20}$$

On the basis of the auxiliary vector $\underline{u}$ - see equation (15.18) - and tolerance coefficient matrix $\underline{\underline{S}}$ - see equation (15.8) -, the auxiliary vector $\underline{v}$ should be estimated by using scalar-vector function

$$f(\underline{v}) = (\underline{u} - \underline{\underline{S}}\underline{v})^2 \qquad . \tag{15.21}$$

Then the vector of the expected values required of internal parameters should be determined by equation

$$\underline{\tilde{x}} = \underline{v} + \underline{\underline{X}}\underline{e}_p \qquad . \tag{15.22}$$

The vectors of minimum and maximum values of the required manufacturing tolerance zones of internal parameters:

$$\underline{x}_{\min} = \underline{\tilde{x}} - 3\underline{\hat{x}} \tag{15.23}$$

and

$$\underline{x}_{\max} = \underline{\tilde{x}} + 3\underline{\hat{x}} \qquad . \tag{15.24}$$

These data have to be investigated from the technological and the manufacturing point of view. If the technological possibilities do not meet the required quality, on the basis of the practicable tolerance zones of the internal parameters should be determined and the base investigation should be performed once more while the external system parameters will meet the requirements. „Use the inverse method of the inverse method." It is also important to mention that this method does not give the unambiguous solution of the technical problem mentioned above. Because it uses any estimation process. This method is „only" an effective adjuvancy to determine the most practicable manufacturing tolerances of the internal parameters during the design of the system.

15.4. Case of Unknown Distributions

The next step is supposing that the measures of made elements of the examined system have unknown distributions, but they are independent random variables.

Then the expected ($\underline{\tilde{x}}$), maximum ($\underline{x}_{max}$) and minimum values ($\underline{x}_{min}$) of internal parameters should be determined and arranged into vectors.

Using the equations (15.10) - (15.13), the expected values of external parameters can be determined.

To determine the possible maximum and minimum values of external parameters vectors of relative values of the internal ones should be determined:

$$\delta \underline{x}_{\max} = \underline{\underline{X}}^{-1}(\underline{x}_{\max} - \underline{\underline{X}}\underline{e}_p) \qquad , \qquad (15.25)$$

$$\delta \underline{x}_{\min} = \underline{\underline{X}}^{-1}(\underline{x}_{\min} - \underline{\underline{X}}\underline{e}_p) \qquad . \qquad (15.26)$$

The so called „positive diagnostic matrix" and „negative diagnostic matrix" should be introduced. Elements of first the one are the positive-sign elements of the original diagnostic matrix - or zero:

$$\underline{\underline{D}}_+ = \left[d_{ij+} = \begin{cases} d_{ij} & if \quad d_{ij} \geq 0 \\ 0 & if \quad d_{ij} < 0 \end{cases} \right] \qquad . \qquad (15.27)$$

Another one's elements are negative-sign elements of the original diagnostic matrix - or zero:

$$\underline{\underline{D}}_- = \underline{\underline{D}} - \underline{\underline{D}}_+ \qquad . \qquad (15.28)$$

Knowing the above mentioned matrices, the vectors of relative maximum and minimum values of the external parameters:

$$\delta \underline{y}_{\max} = \underline{\underline{D}}_+ \delta \underline{x}_{\max} + \underline{\underline{D}}_- \delta \underline{x}_{\min} \qquad , \qquad (15.29)$$

$$\delta \underline{y}_{\min} = \underline{\underline{D}}_+ \delta \underline{x}_{\min} + \underline{\underline{D}}_- \delta \underline{x}_{\max} \qquad . \qquad (15.30)$$

That is, using equations (15.25) and (15.26) :

$$\delta \underline{y}_{\max} = \underline{\underline{X}}^{-1}\left[\underline{\underline{D}}_+ (\underline{x}_{\max} - \underline{\underline{X}}\underline{e}_p) + \underline{\underline{D}}_- (\underline{x}_{\min} - \underline{\underline{X}}\underline{e}_p) \right] \qquad , \qquad (15.31)$$

$$\delta \underline{y}_{\min} = \underline{\underline{X}}^{-1}\left[\underline{\underline{D}}_+ (\underline{x}_{\min} - \underline{\underline{X}}\underline{e}_p) + \underline{\underline{D}}_- (\underline{x}_{\max} - \underline{\underline{X}}\underline{e}_p) \right] \qquad . \qquad (15.32)$$

Then use the nominal values matrix of external parameters and the *k*-dimensional summation vector, the vector of the maximum values of external parameters:

$$\underline{y}_{\max} = \underline{\underline{X}}^{-1}\underline{\underline{Y}}\left[\underline{\underline{D}}_+ (\underline{x}_{\max} - \underline{\underline{X}}\underline{e}_p) + \underline{\underline{D}}_- (\underline{x}_{\min} - \underline{\underline{X}}\underline{e}_p) \right] + \underline{\underline{Y}}\underline{e}_k \qquad , \qquad (15.33)$$

and the vector of their minimum values:

$$\underline{y}_{\min} = \underline{\underline{X}}^{-1}\underline{\underline{Y}}\left[\underline{\underline{D}}_+ (\underline{x}_{\min} - \underline{\underline{X}}\underline{e}_p) + \underline{\underline{D}}_- (\underline{x}_{\max} - \underline{\underline{X}}\underline{e}_p) \right] + \underline{\underline{Y}}\underline{e}_k \qquad . \qquad (15.34)$$

In case of unknown distributions the inverse method of the problem cannot be solved. Then it is acceptable that you suppose normal distributions of internal parameters and use the inverse method depicted by the Chapter 15.3. to estimate the required manufacturing tolerances of internal parameters of the designed system.

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